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MODELS OF SEXUALLY TRANSMITTED DISEASES

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ABSTRACT

In the present work a study of the situation that is presented in Brazil with regard to the sexually transmitted diseases is made, indicating those groups of people for which the most common certain diseases. It introduces a system of differential equations that simulates the process of transmission of different diseases, if different cases are studied that derive from the general system; the system is simplified according to the case, a qualitative study of the system of equations is presented and conclusions are drawn regarding the future situation in relation to the number of infested patients.

KEYWORDS: Sexual, model, disease, patient.

1 INTRODUCTION

The issue of sexually transmitted diseases has always been a concern for policymakers, politicians, religious and the general population. So when celebrations, public holidays and other public activities are held, the measures to be taken to avoid contagion that in later times may cause death; or at least treatments that can be prolonged, which affects not only the infested but the whole family in general.

Studies carried out by [17] allowed to develop a mathematical model of the transmission of infectious diseases; this dynamics of the contagion is modeled by means of sexual activity and, here, one makes use of the concept of individuals susceptible to the contagion with the disease.
The authors in [14], developed a mathematical model on the transmission of gonorrhea, where they study in an exhaustive way the behavior of the trajectories of the system that simulates the process in a neighborhood of the equilibrium points, giving conclusions regarding the future of the disease. Giving a method for the identification of coefficients, where we have a series of data corresponding to a given population.

Syphilis is a disease caused by the bacterium treponema pallidum. It has three stages, of which the first two present the greatest symptoms and potentialities of contagion. The last stage often has no symptoms, which can cause an erroneous impression of curing the disease. Syphilis in pregnant women can cause sequelae in the baby.

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The main symptoms of the disease are:

- In the first stage, sores in the sexual organs and in the groin.
- In the second stage, body spots and hair loss.
- After the symptoms disappear in the third stage, the disease can cause paralysis, brain disease and heart problems, which can even lead to death.

The main forms of transmission are through unprotected sex, blood transfusion and from mother to baby in the management or labor, and the main treatment is penicillin.

In a survey conducted by the Department of Sexually Transmitted Diseases (STDs), AIDS and Viral Hepatitis (DDAHV), the Ministry of Health, which contains data of cases of congenital syphilis updated to June 30, 2014. The sources of the data presented were the compulsory notification of cases of congenital syphilis obtained System for Notifiable Diseases Information System (SINAN) and perinatal death records related to congenital syphilis obtained through the Mortality Information System (SIM), as well as the latest study results parturients conducted by the DDAHV in 2010-2011.

From 1998 to 2014, 104,853 cases of congenital syphilis were reported in SINAN in children under one year of age. In 2013 the rate of live births with syphilis was 4.7 per 1000, showing an increase compared to 2004, when it was 1.7. The states with the highest rates were Rio de Janeiro (11.5), Sergipe (11.2) and Ceará and Alagoas (both with 7.7).

In relation to the mortality rate in children under one year was 1241 between 2000 and 2013, among which 536 cases were in the Southeast region. Of this region, approximately 71% of the cases (378) occurred only in Rio de Janeiro.

A summary of the cases of congenital syphilis detected in 2013 shows the reported cases by region: North, with 1064 cases, Northeast with 4417, Southeast with 5907, South with 1566 and Center-West with 751, totaling 13,705 cases of the disease.

Gonorrhea is an infectious bacterial disease transmitted by the bacterium Neisseria gonorrhoeae. Its main routes of transmission are by sexual and perinatal contact. It is one of the oldest known human diseases. It mainly affects the mucous membranes of the lower genital tract, and more rarely, the mucous membranes of the rectum, oropharynx and conjunctiva. Among women with untreated gonorrhea and / or chlamydia (Chlamydia trachomatis), 10 to 40% develop pelvic inflammatory disease (PID). Of these, more than 25% will become infertile.

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There is a shortage of studies on the disease in Brazil, but the Ministry of Health's National Program on Sexually Transmitted Diseases and AIDS estimated for 1994, 860,265 cases of gonorrhea, equivalent to 56% of all registered sexually transmitted diseases, but officially reported only 119,470 cases from 1987 to 1995.

One of the main difficulties with the transmission of the disease is that in many cases women (70% of those infected) have no symptoms. Thus, in the surveys, the largest number of cases occurs in non-white men between 20 and 24 years (probably due to the greater ease in diagnosis). The hybrid capture test can be used to diagnose gonorrhea and chlamydia, but it is only available in some reference laboratories for research.

Newborns of mothers who are sick or have an infection may have gonococcal conjunctivitis due to contamination in the birth canal and there is a risk that the baby may have severely affected eyes and may lead to blindness.

According to WHO (World Health Organization) data, the estimated new cases of chlamydia in 1999 were 92 million, with 9.5 million occurring in Latin America and the Caribbean. In the US Centers for Disease Control and Prevention (CDC) estimates, there are more new cases of Chlamydia detected than any other STD. In Brazil, estimates from the National STD / AIDS Coordination indicate that 1,967,200 new cases per year are diagnosed.

Genital herpes is a non-healing virus, transmitted predominantly through sexual contact, including through oral sex. The transmission can also be by direct contact with lesions or contaminated objects. It is characterized by the appearance of vesiculous lesions that, in a few days, become small ulcers, preceded by symptoms of burning, pruritus and pain. It is believed that most cases of transmission occur from people who do not know they are infected or are asymptomatic. More recently, the importance of herpes in genital ulcers has been recognized, accounting for a large percentage of cases of HIV transmission, which puts herpes control a priority.

Herpes Simplex Virus (HSV), types 1 (perioral lesion) and 2 (genital lesion) belong to the family Herpesviridae. It may not produce symptoms. In humans, it is more frequently found in the glans and foreskin; in the woman, in the small lips, clitoris, big lips, furcula and cervix. General symptoms such as fever and malaise may occur. With or without symptomatology, after primary infection, HSV ascends the sensory peripheral nerves, penetrates the nuclei of the ganglion cells and enters latency. After primary genital infection by HSV 2 or HSV 1, respectively, 90% and 60% of patients develop new episodes in the first 12 months, by reactivation of the virus.

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The local treatment consists of: physiological solution or 3% boricated water to clean the lesions. There is no treatment for definitive cure of genital herpes, but antivirals reduce the duration of the crisis and prevent relapses, and reduce vertical and horizontal transmission. In pregnant women with herpes simplex, the risk of obstetric complications should be considered. Even in the asymptomatic form, the virus can be transmitted through the birth canal. It is therefore recommended that a cesarean section be performed whenever active herpetic lesions are present.

There are currently over 200 types of HPV (Human Papillomavirus), which can cause genital warts, cancers of the cervix, vagina, vulva, anus and penis. The main form of transmission of HPV is through sexual intercourse. Although more rarely, HPV can be transmitted during childbirth or even by certain objects.

HPV types 16 and 18 cause the majority of cases of cervical cancer worldwide (about 70%). They are also responsible for up to 90% of cases of cancer of the anus, up to 60% of cancers of the vagina and up to 50% of cases of vulvar cancer. Types 6 and 11, on the other hand, cause approximately 90% of genital warts, one of the most common and growing health problems worldwide, and about 10% of low grade cervical lesions.

The main symptoms of warts are irritation or itching on the spot and the risk of transmission is greater when the warts are visible. The lesions may appear on the penis, anus, vagina, cervix, mouth and throat. In many cases, the virus may become latent in the body, disappearing a few days after contact.

The Ministry of Health has adopted the quadrivalent vaccine, which protects against low-risk HPV types 6 and 11, which cause anogenital warts, and high-risk types 16 and 18 that cause cervical cancer. The vaccine, which is preventive, works by stimulating the production of antibodies specific for each type of HPV. Protection against infection will depend on the amount of antibodies produced by the vaccinated person, the presence of these antibodies at the site of infection and their persistence over a long period of time. The HPV vaccine priority population is for girls aged 9 to 13 years, who will receive two doses (0 and 6 months) with a six-month interval, and women living with HIV in the age group of 9 to 26 years, who will receive three doses (0, 2 and 6 months). The HIV vaccine is part of the National Vaccination Calendar and should therefore be available in the routine actions of the Basic Health Units for adolescents and women living with HIV included in the age group served.

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Around the planet, there are around 600 million people infected. Between 75% and 80% of the population get one or more types of HPV at some point in life. According to the World Health Organization (WHO), STDs are among the top ten causes of demand for health services in the world.

The problem of the modeling of sexually transmitted diseases has attracted the attention of researchers from different epochs, appearing models, gonorrhea, syphilis among other diseases that were modeled by systems of differential equations. Among these models the following works may be indicated, [1], [2], [7], [6], [8], [10], [12], [13] and [11].

In [9] and [5], respectively, the ecological problem and the natural history of the disease are studied.

Modeling of HIV has also been frequently dealt with in the specialized literature. An example of this is [4] and [3].

The treatment that we will make in this case corresponds to other models presented in the researches of other diseases, especially the case of sicklemia, quite treated and with many models already developed, will only mention some of these works. In [15] and [16} is the qualitative study of different models in an autonomous and non-autonomous form of polymer formation.

The following is a similar model to that presented in [14], indicating certain conclusions regarding the disease as a function of the coefficients of the model.

II. FORMULATION OF THE MODEL

Consider that x(t) represents the number of sexually active and infected women, and y(t) to the amount of sexually active and infected men, M the total number of sexually active women, and H it is clear that in order for new infected women to appear. There must be contact between a healthy woman and an infected man, and for the appearance of new infected men there must be a meeting of a healthy man with an infected woman. Moreover, without such encounters, both sick and ill women over time will be reduced, thus the following system will be used to simulate this contagion of the disease,

$$\begin{cases} \dfrac{dx}{dt} = -ax + b(M - x)y \\ \dfrac{dy}{dt} = -cy + d(H - y)x \end{cases} \tag{1}$$

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With the initial condition, is to say if it considers that in the initial moment the populations of infested women and men are respectively, and. It is evident that the region where the system will be studied as the form,

$$\pi = \left\{(x, y) \in R^2 \,/\, 0 \le x \le M, 0 \le y \le H\right\}$$

The constants a, b, c, d, H and M used in the system are positive, which; represent a and c the rate of recovery for both sexes, and b and d the repetitive rates of appearance of new women and new men infected with the disease.

The previous system can be expressed as follows,

$$\begin{cases} \dfrac{dx}{dt} = -ax + bMy - bxy \\ \dfrac{dy}{dt} = dHx - cy - dyx \end{cases} \qquad (2)$$

The matrix of the linear part of the system has the form,

$$A = \begin{pmatrix} -a & bM \\ dH & -c \end{pmatrix}$$

We are interested in the qualitative study of the behavior of the system trajectories in a neighborhood of the equilibrium positions of the system, ie the points $(0,0)$ and (h,k) determined by the system solutions,

$$\begin{cases} -ax + bMy - bxy = 0 \\ dHx - cy - dxy = 0 \end{cases}$$

Thus,

$$h = \frac{bdMH - ac}{d(a + bH)}, \quad k = \frac{bdMH - ac}{b(c + dM)}.$$

For the study of the behavior of the solutions in a neighborhood of the equilibrium positions, we first need to determine the eigen values of matrix A, since in many cases only this allows us to infer the behavior of the solutions of the system, ie the solutions of the characteristic equation,

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$$\begin{vmatrix} -a-\lambda & bM \\ dH & -c-\lambda \end{vmatrix} = 0$$

This equation is equivalent to the equation of the second degree,

$$\lambda^2 + (a+c)\lambda + ac - bdHM = 0$$

In order to carry out this study we will separate from the general problem the following particular cases, as a function of the determinant signal of the matrix A corresponding to the linear part of the system.

III. When the determinant of the linear part matrix is positive.

In this case, in the region considered, there is only the equilibrium position ; to make an exhaustive study the system (2) will be reduced to a simpler form.

Theorem 1: There is the nondegenerate linear transformation,

$$\begin{cases} x = 2bMx_1 + (c-a-\sqrt{\Delta})x_2 \\ y = (a-c+\sqrt{\Delta})x + 2bHx_2 \end{cases} \quad (3)$$

which reduces the system (2) to the system,

$$\begin{cases} x_1^{'} = \lambda_1 x_1 + M_1 x_1^2 + N_1 x_1 x_2 + L_1 x_2^2 \\ x_2^{'} = \lambda_2 x_2 + M_2 x_1^2 + N_2 x_1 x_2 + L_2 x_2^2 \end{cases} \quad (4)$$

On de $\Delta = (a-c)^2 + bdMH$.

Demonstration: To arrive at the proof of this theorem, we derive the transformation (3) along the trajectories of systems (2) and (4). Evidently

λ_1 and λ_2 they are real because $\Delta > 0$, and the parameters appearing in the system (4) have the form,

$$\lambda_1 = -\frac{a+c-\sqrt{\Delta}}{2},\ \lambda_2 = -\frac{a+c+\sqrt{\Delta}}{2},$$

$$M_1 = 2Mb^2(c-a-\sqrt{\Delta}),\ M_2 = 2Mbd(c-a-\sqrt{\Delta}),$$

$$N_1 = 2b[(a^2+c^2+(a-c)\sqrt{\Delta} - 2bdac],\ N_1 = 2d[(a^2+c^2+(a-c)\sqrt{\Delta} - 2bdac],$$

$$L_1 = 2bdH[a-c+\sqrt{\Delta}],\ L_2 = 2d^2H[a-c+\sqrt{\Delta}].$$

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One can study the behavior of the solution in a neighborhood of the point $(0,0)$ from the signal of the eigenvalues of the matrix A, that is, by applying the first approximation method.

As $\det A = ac - bdMH$ is positive the two eigenvalues of the matrix are negative, ie the equilibrium position $(0,0)$ is asymptotically stable.

IV. When the determinant of the linear part matrix is negative.

In this case in the region considered there are two equilibrium positions $(0,0)$ and (h,k); with what, $ac < bdMH$ system (2) has a positive own value and a negative value, then by the method of the first approximation one has that the origin is an unstable equilibrium position.

To analyze the behavior of the trajectories at the point (h,k) and to reach conclusions regarding the behavior of the disease, it is necessary to translate the coordinate origin to the position (h,k), thus by means of the transformation of coordinates,

$$\begin{cases} x = y_1 + h \\ y = y_2 + k \end{cases}$$

If the system (4) is reduced to the system,

$$\begin{cases} y_1^{'} = -(a+bk)y_1 + (bM - bh)y_2 - by_1y_2 \\ y_2^{'} = (dH - dk)y_1 - (c+dh)y_2 - dy_1y_2 \end{cases} \quad (5)$$

Conclusion: If $\det A > 0$**,**then in the region $\pi = \{(x,y) \in R^2 / 0 \le x \le M, 0 \le y \le H\}$ there is only one equilibrium position $(0,0)$ and by the method of the first approximation is asymptotically stable and therefore the number of patients will decrease with time.

Theorem 2: There is the no degenerate linear transformation,

$$\begin{cases} y_1 = 2(bM - bh)z_1 + (c + dh - a - bk - \sqrt{\Delta^{'}})z_2 \\ y_2 = (a + bk - c - dh + \sqrt{\Delta^{'}})z_1 + 2(dH - dk)z_2 \end{cases} \quad (6)$$

which reduces the system (5) to the system,

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$$\begin{cases} z_1^{'} = \lambda_1^{'} z_1 + M_1^{'} z_1^2 + N_1^{'} z_1 z_2 + L_1^{'} z_2^2 \\ z_2^{'} = \lambda_2^{'} z_2 + M_2^{'} z_1^2 + N_2^{'} z_1 z_2 + L_2^{'} z_2^2 \end{cases} \quad (7)$$

At where $\Delta^{'} = (a - c + bk - dh)^2 + 4bd(H - k)(M - h)$.

Demonstration: In order to check the proof of this theorem we derive the transformation (6) along the trajectories of systems (5) and (7).

The matrix of the linear part of the system (5) has the shape,

$$A' = \begin{bmatrix} -(a + bk) & bM - bh \\ dH - dk & -(c + dh) \end{bmatrix}$$

The characteristic equation for determining the eigenvalues of the matrix has the form,

$$\lambda^2 + (abk + c + dh)\lambda + (a + bk)(c + dh) - bd(h - M)(k - H) = 0,$$

As $\Delta > 0$ the eigenvalues of the matrix A are real, and thus one has to that the parameters that appear in the system (7) have the form,

$$\lambda_1^{'} = -\frac{a + c + bk + dh - \sqrt{\Delta^{'}}}{2},\ \lambda_2^{'} = -\frac{a + c + bk + dh + \sqrt{\Delta^{'}}}{2},$$

$$M_1^{'} = 2b(bM - bh)(c + dk - a - bh - \sqrt{\Delta^{'}}),\ M_2^{'} = 2d(bM - bh)(c + dk - a - bh - \sqrt{\Delta^{'}}),$$

$$N_1^{'} = 2b[(a + bk)^2 + (c + dh)^2 + (a + bk - c - dh)\sqrt{\Delta} - 2(bM - bh)(dH - dk)],$$

$$N_2^{'} = 2d[(a + bk)^2 + (c + dh)^2 + (a + bk - c - dh)\sqrt{\Delta} - 2(bM - bh)(dH - dk)],$$

$$L_1^{'} = 2b(dH - dk)(a + bk - c - dh + \sqrt{\Delta}),\ L_2^{'} = 2d(dH - dk)(a + bk - c - dh + \sqrt{\Delta}).$$

In this case as the determinant of the matrix of the linear part is negative by the conditions of Hurwitz for a second order system it can be concluded that the two eigenvalues are negative and by the first approximation method the equilibrium position (h, k) is asymptotically stable.

V. CONCLUSIONS

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- If $\det A > 0$ then in the region $\pi = \{(x, y) \in R^2 / 0 \le x \le M, 0 \le y \le H\}$ there is only one equilibrium position, the point $(0,0)$ which is asymptotically stable, and therefore the disease will decrease over time.

- If $\det A < 0$ then in the region $\pi = \{(x, y) \in R^2 / 0 \le x \le M, 0 \le y \le H\}$ there are two equilibrium positions $(0,0)$ which is unstable and (h, k) which is asymptotically stable if $\det A' > 0$, and therefore if in this case (h, k) is sufficiently close to the origin of quantities of patients will not be too many numbers, but if this point is too far from the origin, prophylactic measures necessary to prevent this disease to become an epidemic must be taken.

- If and then the point is unstable, and therefore prophylactic measures must be taken to reverse this situation, owing to the rapid increase in people infected with the disease.

Note: Here we have treated the cases where, the case is of paramount importance, but because it is a critical case it will be necessary to make use of the Analytical Theory of Differential Equations to simplify the system and reach conclusions.

VI. BIBLIOGRAPHY

1. Bailey N. T. J., The Mathematical Theory of Infectious Diseases, 2nd ed., Hafner, New York,1975 (1).
2. Bailey N. T. J, The Mathematical Theory of Infectious Diseases, 2nd ed., Hafner, New York,1975 (2).
3. Becker N., Analysis of Infectious Disease Data, Chapman and Hall, New York, 1989.
4. Bongaarts J., A model of the spread of HIV infection and the demographic impact of AIDS,Stat. Med., 8 (1989), pp. 103–120.
5. Burnett M. and White D. O., Natural History of Infectious Disease, 4th ed., Cambridge University Press, Cambridge, UK, 1974.
6. Dietz K. and Schenzle D., Mathematical models for infectious disease statistics, in A Cele-bration of Statistics, A. C. Atkinson and S. E. Feinberg, eds., Springer-Verlag, New York,1985, pp. 167–204.
7. Dietz K., The incidence of infectious diseases under the influence of seasonal fluctuations in Mathematical Models in Medicine, J. Berger, W. Buhler, R. Repges, and P. Tautu, eds.,Lecture Notes in Biomath. 11, Springer-Verlag, Berlin, 1976, pp. 1–15.
8. Earn D. J. D., Rohani P., Bolker B. M., and Grenfell B. T., A simple model for complex dynamical transitions in epidemics, Science, 287 (2000), pp. 667–670.
9. Grenfell B. T. and Dobson A. P., eds., Ecology of Infectious Diseases in Natural Populations, Cambridge University Press, Cambridge, UK, 1995.

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10. Gao L. Q. and Hethcote H. W., Disease transmission models with density-dependent demo-graphics, J. Math. Biol., 30 (1992), pp. 717–731.
11. Herbert W. and Hethcote H. W., The Mathematics of Infectious Diseases. Society for Industrial and Applied Mathematics. Vol. 42, No. 4, pp. 599–653 (2000).
12. Hethcote H. W., Modeling heterogeneous mixing in infectious disease dynamics, in Models for Infectious Human Diseases, V. Isham and G. F. H. Medley, eds., Cambridge University Press, Cambridge, UK, 1996, pp. 215–238.
13. Hethcote H. W., Qualitative analyses of communicable disease models, Math. Biosci., 28 (1976), pp. 335–356.
14. Repilado, J. A., Ruiz, I. A., & Bernal, A. Analysis and identification of a mathematical model of the transmission of infectious diseases. (1998). Cienc. Mat., 16 (1), 65 - 69.
15. Sánchez, Sandy; Ruiz, Antonio I.; Fernández, Adolfo. “A model of the molecular processes of the polymerization and crystallization of hemoglobin S ", Rev. Mathematical Sciences. 26 (2012), No. 1, pp 53-57.
16. Sánchez, S., Fernández, G. A. A., Ruiz. A. I., & Carvalho, E. F.” Model of periodic coefficients in the polymerization function ".
Tecnica Vitivinicola Journal, (2016).
17. Velasco, J. X. Mathematical Models in Epidemiology: approaches and
reaches. (2007). Journal of Mathematics, 44, 11 - 27.