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CYCLICAL BEHAVIOR OF GAS INTERCHANGE BETWEEN LUNGS AND THE MIDDLE ENVIRONMENT

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ABSTRACT

In this article a general study is made about the lungs, their characteristics, their main function, detailing some aspects of the respiratory process; the main pulmonary diseases are indicated and how to prevent them. A model is made by means of a system of differential equations that simulates the blood oxygenation process, a qualitative study is made, and conclusions are given regarding the functioning in a healthy person; in the critical case of a pair of pure imaginary eigenvalues, the system is reduced to the normal form to facilitate qualitative study, where the cyclical behavior of the process is seen. Examples are given, the quays show the results obtained in a theoretical way, indicating the validity of the simulation with the real problem.

KEYWORDS: Lung, qualitative study, cyclical process, breathing

I. INTRODUCTION

The lungs are spongy and elastic organs formed by millions of air-filled alveoli. It is approximately 25 cm long and 700 g in weight. The right lung is larger in width than the left, as it has three lobes, one more than the left, but it is shorter in height, because on the right side the liver is present, causing the diaphragm to be higher. In the left lung there is a cardiac notch.

The lungs are attached to the pericardium through pulmonary ligaments and to the trachea and heart by structures called the hilum, comprising pulmonary vessels, lymphatic vessels, bronchial vessels, main bronchi and nerves that arrive and leave the lungs. The lungs are covered by a thin layer, the pleura that consists of a transparent and thin membrane. The inner pleura is attached to the pulmonary surface, and the outer pleura is attached to the wall of the rib cage. In the intermediate space of the pleura there is a small space, occupied by a lubricating liquid secreted by the pleura, this liquid is what holds the two pleuras together, due to surface tension, causing them to slide during breathing movements [9], [10].

The base of each lung rests on the diaphragm, an organ that separates the chest from the abdomen, present only in mammals, promoting, together with intercostal muscles, respiratory movements. In the lungs, the bronchi ramify intensely, giving rise to increasingly thin tubes, the bronchioles. The highly branched set of bronchioles is the bronchial tree or respiratory tree.

In pulmonary breathing, air enters and leaves the lungs due to contraction and relaxation of the diaphragm. When the diaphragm contracts, it decreases the pressure in the lungs and the air outside the body enters rich in oxygen; process called inspiration. When the diaphragm relaxes, the pressure inside the lungs increases and the air that was inside now comes out with carbon dioxide; process gives rise to a cyclical process, as this exchange of gases between the lungs and the environment periodically takes place. This would be an essential way to detect lung diseases, including the coronavirus [17].

People can stop breathing but no one can stop breathing for more than a few minutes, because the concentration of carbon dioxide in the blood gets so high that the body can no longer supply energy to the cells and the bulb, part of the nervous system that it forms the brain, sends nerve impulses to the diaphragm and intercostal muscles, so that they contract and breathing is resumed normally.

The person may suffer from different lung diseases such as, Bronchitis, Tuberculosis, Pulmonary emphysema, Pneumonia, Asthma, Lung cancer, etc. When inflammation occurs in an individual's lungs, more specifically in the alveoli, we call it pneumonia, due to infection caused by bacteria, viruses, fungi and other infectious agents. Pneumonia can cause death if left untreated; these diseases can damage the pulmonary alveoli, decreasing the lung's ability to perform its function [4], [11], [13], [14].

The main purpose of the lungs is to supply our blood with oxygen, which is transported to the cells of the body. The other respiratory organs have the function of directing the air to the lungs, it is in them that the conversion of venous blood, blood low in oxygen and rich in carbon dioxide, into arterial blood, blood rich in oxygen occurs. When we breathe, we start a complex path, the air enters through the nose, or through the mouth, goes to the trachea following small tubes, the bronchi. From the bronchi, air is taken to other pulmonary regions; an involuntary movement that is controlled by the brain controls the entry and exit of air from the lungs.

Respiratory movement is controlled by a nerve center located in the spinal cord; under normal conditions, this center produces an impulse every 5 seconds, stimulating the contraction of the thoracic muscles and the diaphragm, where we inhale. However, when the blood becomes more acid due to the increase in carbon dioxide, the medullary respiratory center induces the acceleration of respiratory movements.

In the event of a decrease in the concentration of oxygen gas in the blood, the respiratory rate is also increased; this reduction is detected by chemical receptors located on the walls of the aorta and the carotid artery; however, when the air enters or leaves the organism through the mouth, however, the moistening and heating of the air is incomplete without the filtration of particles of dust, smoke, and even microscopic living beings, such as viruses and bacteria, capable of causing damage to our health. Some impurities are "filtered" in different organs of the respiratory system, but others can pass to the lungs, causing diseases.

Human beings have neurons in the bulb region that guarantee the regulation of breathing. The bulb perceives changes in the pH of the surrounding tissue liquid and triggers responses that guarantee changes in the respiratory rhythm. When carbon dioxide levels rise in the blood and cerebrospinal fluid, a drop in pH occurs. This happens because the carbon dioxide present in these places can react with water and trigger the formation of carbonic acid.

The bulb then notices these changes, signals are sent to the intercostal muscles and diaphragm to increase the intensity and rate of breathing. When the pH returns to normal, there is a reduction in respiratory rate and intensity. It is worth noting that changes in the level of oxygen in the blood trigger few effects on the bulb. However, when the levels are very low, the breathing rate increases. [12].

Several works, books and articles related to processes in human life are known; among these books dedicated to mathematical modeling, the following are indicated [6], [7] and [8] in which real problems are simulated by means of equations and systems of differential equations, where in addition a certain treatment is done to give conclusions of the processes. In [8] the authors simulate the process of polymer formation in the blood using autonomous systems of differential equations of third and fourth order, giving conclusions on the formation of polymers and domains.

In [7] different problems of real life are treated by means of equations and systems of differential equations, all of them only in the autonomous case; where examples are further developed, and problems and exercises are placed so that they can be developed by the reader. The authors of [6] indicate a set of articles forming a collection of several problems that are modeled in different ways, but in general the qualitative and analytical theory of differential equations is used in both autonomous and non-autonomous cases.

A compartment system essentially consists of a finite number of interconnected subsystems, called compartments, which exchange between and with the environment, amount of concentration of materials or substances, each compartment is defined by its physical properties; in particular, the dynamics of a drug in the human body were treated; not all drugs have the same route, but in the ingestible case in [1], sufficient conditions are given for their elimination; the case of an inhalable drug is treated in [3] and injectable in [2], in all cases after the qualitative study of the system used in the modeling the future situation of the process is predicted.

Insulin is a hormone produced by the pancreas; its function is to act in the reduction of glycemia (blood glucose rate). It is responsible for the absorption of glucose by cells; when insulin-glucose dynamics are not natural in the human body, diabetes can be produced, this dynamics in both a normal and diabetic person is modeled in [15] and [16]. The case of tissue replacement is simulated in [5], the case of diabetic foot is treated.

Studies carried out in [21] have allowed the development of a mathematical model for the transmission of infectious diseases; this dynamic of contagion is modeled by means of sexual activity and, here, the concept of individuals susceptible to contagion with the disease is used.

Pollution elimination problems are other models that have been treated by several authors, in particular cases with periodic coefficients [20]; this type of modeling is also used in [18] and [19], as it has the characteristics that best fit the real problem due to its characteristics.

II. MODEL FORMULATION

In order to simulate the process of blood oxygenation through the lungs, it is necessary to take into account some basic principles regarding this process, firstly that the lungs do not provide more oxygen than our body needs, so in their variation will increase proportionally to the concentration of carbon dioxide and decrease proportionally to its own concentration; but in the variation of carbon dioxide, it is added proportionally to its concentration and decreased proportionally to the concentration of oxygen.

In order to formulate the model using a system of differentiable equations, the following variables will be introduced:

-$\tilde{x}_1$ is the total concentration of oxygen in the blood at the moment t.
-$\tilde{x}_2$ is the total concentration of carbon dioxide in the lungs at the moment t.
In addition, $\bar{x}_1$ and $\bar{x}_2$ the optimal values of oxygen and carbon dioxide in the lungs respectively.

Here the variables will be introduced x_1 and x_2 defined as follows: $x_1 = \tilde{x}_1 - \bar{x}_1$ and $x_2 = \tilde{x}_2 - \bar{x}_2$ so, if $\bar{x}_1 \to 0$ and $\bar{x}_2 \to 0$ the following conditions would be met $\tilde{x}_1 \to \bar{x}_1$ and $\tilde{x}_2 \to \bar{x}_2$, which would constitute the main objective of this work. So, the model will be given by the following system of equations,

$$\begin{cases} x_1' = -a_1x_1 + a_2x_2 + X_1(x_1,x_2) \\ x_2' = -a_3x_1 + a_4x_2 + X_2(x_1,x_2) \end{cases} \quad (1)$$

Where $X_i(x_1,x_2)$, $(i = 1,2)$ they are disturbances not inherent in the process, which could at a given moment produce certain changes; and from a mathematical point of view they are infinitesimals of a higher order, those that admit the following development in series of potentials,

$$X_i(x_1,x_2) = \sum_{|p|\geq 2} X_i^p x_1^{p_1} x_2^{p_2} \; (i = 1,2), |p| = p_1 + p_2$$

The parameters present in the system (1), have the following meaning,

a_1- It represents the coefficient of decrease in oxygen concentration as a function of its concentration.

a_2- It represents the coefficient of increase in the concentration of oxygen as a function of the concentration of carbon dioxide.

a_3- Represents the coefficient of decrease in the concentration of carbon dioxide as a function of its concentration.

a_4- It represents the coefficient of increase in the concentration of carbon dioxide as a function of the concentration of oxygen.

The characteristic equation of the matrix of the linear part of the system (1) has the following form,

$$\begin{vmatrix} -a_1 - \lambda & a_2 \\ -a_3 & a_4 - \lambda \end{vmatrix} = 0$$

This expression is equivalent to,

$$\lambda^2 + (a_1 - a_4)\lambda + (a_2a_3 - a_1a_4) = \mathbf{0} \quad \textbf{(2)}$$

In this case, applying the first approximation method, the following result is obtained.

Theorem1: The null solution of the system (1) is asymptotically stable if and only if the following conditions are met: $a_1 > a_4$ and $a_2a_3 > a_1a_4$, otherwise, it is unstable.

The proof is a direct consequence of the conditions of the Hurwitz theorem, because in this case both n_1 and n_2 are positive, this responds to the asymptotic stability theorem of Liapunov's second method.

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Suppose that $a_1 = a_4$ and $a_2 a_3 > a_1 a_4$, , in this case the method of first approximation is not applicable because it has a double of pure imaginary eigenvalues; by means of a non-degenerate transformation $X = SY$, the system (2) can be transformed into the system,

$$\begin{cases} y_1' = \sigma i\, y_1 + Y_1(y_1, y_2) \\ y_2' = -\sigma i y_2 + Y_2(y_1, y_2) \end{cases} \quad \textbf{(3)}$$

Nesse caso será aplicado o segundo método de Liapunov uma vez reduzido esse sistema à forma normal.

Teorema2: A mudança de variáveis,

$$\begin{cases} y_1 = z_1 + h_1(z_1, z_2) \\ y_2 = z_2 + h_2(z_1, z_2) \end{cases} \quad \textbf{(4)}$$

transforma o sistema (3) na forma normal,

$$\begin{cases} z_1' = \sigma i z_1 + z_1 P(z_1 z_2) \\ z_2' = -\sigma i z_2 + z_2 \overline{P}_2(z_1 z_2) \end{cases} \quad \textbf{(5)}$$

Where $h_i, i = 1,2$ P and $\bar{P}$ are power series like X_i.

Proof: Derivando a transformação (4) ao largo das trajetórias dos sistemas (3) e (5) se obtém o sistema de equações,

$$\begin{cases} (p_1 - p_2 - 1)\sigma i h_1 + z_1 P = Y_1 - \frac{\partial h_1}{\partial z_1} z_1 P - \frac{\partial h_1}{\partial z_2} z_2 \overline{P} \\ (p_1 - p_2 + 1) i\sigma h_2 + z_2 \overline{P} = Y_2 - \frac{\partial h_2}{\partial z_1} z_1 P - \frac{\partial h_2}{\partial z_2} z_2 \overline{P} \end{cases} \quad \textbf{(6)}$$

The system (6) allows to determine the series coefficients, h_1, h_2, P and $\overline{P}$; the resonance equations are, $p_1 - p_2 - 1 = 0$ and $p_1 - p_2 + 1 = 0$; with what P and $\overline{P}$ are different from zero in the resonant case, being h_1 and h_2 equal to zero in this case, as they are arbitrary applying uniqueness if they become equal to zero, and by the resonance equations it is deduced to forms of the powers of P and $\overline{P}$. In the non-resonant case, the series P and $\overline{P}$ null and in this case h_1 and h_2 are uniquely determined.

The series P and $\overline{P}$ admit the following development in power series,

$$\boldsymbol{P(z_1 z_2) = \sum_{n=k}^{\infty} a_n (z_1 z_2)^n + i \sum_{n=l}^{\infty} b_n (z_1 z_2)^n}$$

Theorem3: If $a_k < 0$, then the trajectories of the system (5) are asymptotically stable, otherwise they are unstable.

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Proof: Consider the Lyapunov function defined positive,

$V(z_1, z_2) = z_1 z_2$

The function V is such that its derivative along the trajectories of the system (5) has the following expression,

$V'(z_1, z_2) = a_k (z_1 z_2)^{k+1} + R(z_1, z_2)$

It can be seen that the derivative $V'(z_1, z_2)$ is defined negative, because in the function $R(z_1, z_2)$ you only have terms of a degree greater than $2(k+1)$. This coupled with that $V(z_1, z_2)$ is positive and guarantees the asymptotic stability of the null solution of the system (5).

Observation: If it appears that the solutions in the system (5) have a cyclic form, that is to say they constitute periodic solutions, as shown in the system that appears below.

$$\begin{cases} z_1' = \sigma z_2 + z_1 P[z_1 z_2\{(z_1 + i z_2)(z_1 - i z_2) - 1\}] \\ z_2' = -\sigma z_1 + z_2 \bar{P}[z_1 z_2\{(z_1 + i z_2)(z_1 - i z_2) - 1\}] \end{cases}$$

The functions $z_1 = sen\sigma t$, $z_2 = cos\sigma t$ they are periodic solutions of the previous system; and the eigenvalues of the matrix of the linear part of the system are $\lambda_1 = i\sigma$ and $\lambda_2 = -i\sigma$, what corresponds with the studied problem. This is in correspondence with blood oxygenation, a process that is done periodically, the simulation corresponds with what appears in real life.

In correspondence with the sign of the coefficient of the real part of the first nonzero term of the function P, a spiral could appear that could enter or leave the coordinate origin; the following as an illustration the following two cases are applied,

$$\begin{cases} z_1' = -z_1 + 3z_2 - 3z_1^3 z_2^2 \\ z_2' = -2z_1 + 2z_2 - 2z_1^2 z_2^3 \end{cases} \quad (7)$$

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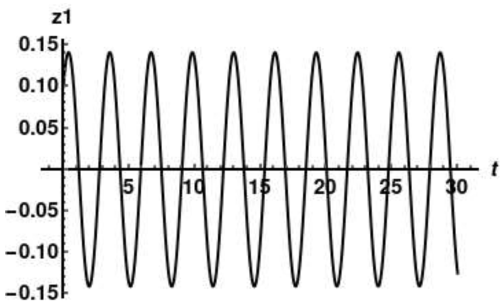

Fig.1. Graph of z_1 in time.

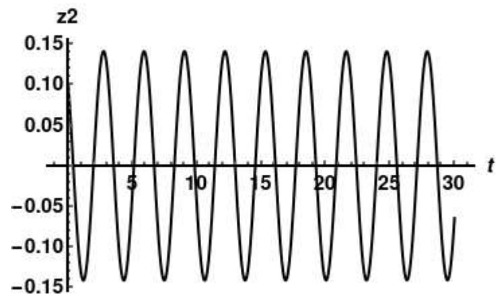

Fig.2. Graph of z_2 in time.

And in this case for the system (7) the graph of z_1 concerning z_2 has shape,

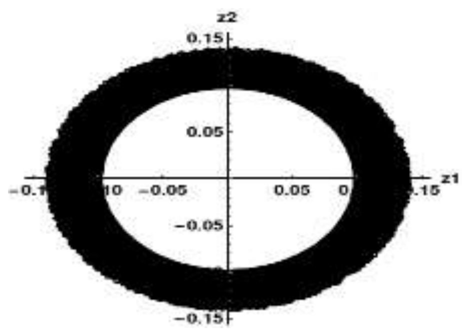

Fig.3. Graph of z_1 concerning z_2.

From this it can be deduced that there is still a lot to digress if there is a spiral that converges towards the origin of the coordinate; this corresponds with what has been demonstrated before theoretically.

III. CONCLUSIONS

1. The problem of simulating the blood oxygenation process by means of the lungs is a topical and transcendental problem, because this way, conditions can be provided for a comfortable process.
2. Theorem1 provides necessary and enough conditions for the total concentration of oxygen and the total concentration of carbon dioxide in the lungs to converge to allowable values making the patient's life comfortable.
3. The critical case of a pair of pure imaginary eigenvalues, considered in this work is possible and that is why their study is important.
4. Theorem 2 allows to transform the system (3) in the normal form, which facilitates the qualitative study to give conclusions regarding the patient's future situation.

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5. If $a_k < 0$, the total concentration of oxygen and the total concentration of carbon dioxide in the lungs converge to allowable values which would give comfort to the person's life; otherwise, urgent measures are needed to change the respiratory process to prevent the patient from entering into crises.
6. Here, it is demonstrated that this is a cyclical problem, a model that simulates the process has periodic solutions; which corresponds to reality, as the exchange of air in the lungs is carried out periodically.
7. The graphed examples allow us to realize that the solutions may be cyclical, but that they may be in the form of a spiral that enter the origin's exit; this indicates the possible pulmonary affect of the person suffering at that moment.

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