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## ADAPTIVE MODE CONTROL FOR BIOMASS ANAEROBIC DIGESTION SYSTEM WITH UNCERTAIN PARAMETERS

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### ABSTRACT

In this paper, a novel mode of biomass anaerobic digestion process with partly uncertain parameters is considered. Based on the Lyapunov stability theorem and linear matrix inequality (LMI), a simple adaptive sliding mode controller for the proposed biomass anaerobic digestion system with partly unknown parameters was concerned. The upper bounds of parameters are completely unknown. Finally, the numerical example results are provided to validate the theoretical and practical merit of the proposed scheme.

**KEYWORDS:** adaptive; biomass anaerobic digestion process; sliding mode control; stability

### 1. INTRODUCTION

Bio energy, also known as green energy, which is the earliest human use of energy, is the energy derived from biomass. Biomass is a variety of organisms that are produced through photosynthesis by using of air, water, land, etc. that is, all organic matter that can grow is called biomass. It includes plants, animals, and microbes.

In the broadest sense, biomass includes all plants, microorganisms and animals, plants, microorganisms, food, and waste materials, for example, crops, crop waste, wood, wood waste and animal manure. In a narrow sense, mainly refers to the agriculture and forestry biomass production process in addition to food, other than fruit stalks, trees and other wood cellulose (referred to as lignin), Agro industrial waste, agricultural waste and animal husbandry production process such as manure and waste materials. Biomass energy has several characteristics: renewable, low pollution, wide distribution, abundant amount of biomass fuels [1-6].

In the face of global reduction of fossil energy consumption, controlling of greenhouse gas emissions situation, the use of biomass energy resources to produce renewable energy products, has become one of the important ways to deal with global climate change and greenhouse gas emission control in the world. Because of its renewable, low pollution, low pollution, wide distribution, rich and so on, it has become an

important option for the alternative energy sources [7-15].

Developed countries and developing countries have a certain difference in the technical level, development mode, quantity scale, commercial degree of the use of biomass energy. As the technology and equipment localization degree is not high, the cost of new energy development and utilization is high, compared to similar products, its market competition ability is weak. Therefore, it is very significant to study the process of anaerobic digestion of biomass ([16-18]).

Motivated by the above discussions, this paper intends to establish a new mode of biomass anaerobic digestion system, and aims to the problem of adaptive sliding mode control design procedure was considered for biomass anaerobic digestion system with uncertain parameters. It is assumed that the upper bounded of the parameters of the proposed biomass anaerobic digestion system are completely unknown in advance. Finally, the numerical example results are provided to validate the theoretical and practical merit of the proposed scheme.

## 2. Mathematical model

In this section, a new mathematical mode of biomass anaerobic digestion system is proposed. Motivated by [9], we first consider the following biomass anaerobic digestion mode:

$$\left\{ \begin{array}{l} \dot{x}_1(t) = -bx_1(t)x_2(t) + (Y_p S_{in} - x_1(t))u(t), \\ \dot{x}_2(t) = \frac{u_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t) - x_2(t)u(t), \\ \dot{x}_3(t) = \frac{k_1 u_{m1} x_3(t)}{k_{s1} + x_3(t)}x_2(t) + bx_1(t)x_2(t) - x_3(t)u(t), \\ \dot{x}_4(t) = \frac{u_{m2}x_5(t)}{k_{s2} + x_5(t)}x_4(t) - x_4(t)u(t), \\ \dot{x}_5(t) = -\frac{k_2 u_{m2} x_5(t)}{k_{s2} + x_5(t)}x_4(t) + \frac{Y_b u_{m1} x_3(t)}{k_{s1} + x_3(t)}x_2(t), \\ \quad \quad \quad -x_5(t)u(t) \end{array} \right. \quad (1)$$

where  $x_1(t), x_2(t), x_4(t), x_6(t)$  and  $S_{in}$  represent the concentrations of the soluble organics, acid-producing bacteria, methanogen, output of methane and fermentation mate respectively,  $x_3(t)$  and  $x_5(t)$  represent the substrate concentrations of the acid-producing bacteria and output of methane.  $S_{in} = S_{i0} - \omega(t)$ ,  $S_{i0} \in [30, 80]$  is a constant,  $\omega(t)$  represents a disturbance.  $u(t)$  is a control term or dilution type.  $b, Y_p, Y_b, k_1, k_2, k_{s1}, k_{s2}, S_{i0}, u_{m1}$  are model unknown parameters. The above parameters were positive.

Yet the general, we assume  $\omega(t) = \delta_1(t)Y_pS_{i0}$ , and  $Y_pS_{in} - x_1(t) \leq 0$ , where  $0 < d_1 \leq \delta_1(t) \leq d_1 < 1$ .

Letting  $d = Y_pS_{i0}$ . It is easy to see that  $d > 0$ . Through simple derivation of system (1), it can obtain the following biomass anaerobic digestion mode

$$\left\{ \begin{array}{l} \dot{x}_1(t) = -bx_1(t)x_2(t) - x_1(t)u(t) + d(1 - \delta_1)u(t), \\ \dot{x}_2(t) = \frac{u_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t) - x_2(t)u(t), \\ \dot{x}_3(t) = \frac{k_1u_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t) + bx_1(t)x_2(t) - x_3(t)u(t), \\ \dot{x}_4(t) = \frac{u_{m2}x_5(t)}{k_{s2} + x_5(t)}x_4(t) - x_4(t)u(t) \\ \dot{x}_5(t) = -\frac{k_2u_{m2}x_5(t)}{k_{s2} + x_5(t)}x_4(t) + \frac{Y_bu_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t) - x_5(t)u(t), \\ \dot{x}_6(t) = -\frac{Y_gk_2k_{s2}(u_{m2}x_4(t))^2x_5(t)}{(k_{s2} + x_5(t))^3} + \frac{Y_gY_bk_{s2}u_{m1}u_{m2}x_2(t)x_3(t)x_4(t)}{(k_{s2} + x_5(t))^2(k_{s1} + x_3(t))} \\ \quad + Y_g\left(\frac{k_2u_{m2}x_5(t)}{k_{s2} + x_5(t)}\right)^2x_4(t) \\ \quad - \left(\frac{Y_gk_{s2}u_{m2}x_4(t)x_5(t)}{(k_{s2} + x_5(t))^2} + \frac{Y_gu_{m2}x_4(t)x_5(t)}{k_{s2} + x_5(t)} + x_6(t)\right)u(t). \end{array} \right. \quad (2)$$

The formula (1) can be transformed to the following matrix equation

$$\begin{aligned} \dot{x}(t) &= (f(x(t)) - x(t))u(t) \\ &+ d(I - \Lambda(t))Bu(t), \end{aligned} \quad (3)$$

where

$$\begin{aligned} x(t) &= (x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6)^T, \\ f(x) &= (f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6)^T, \\ f_1(x) &= -bx_1(t)x_2(t), \\ f_2(x) &= \frac{u_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t), \\ f_3(x) &= \frac{k_1u_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t) + bx_1(t)x_2(t), \end{aligned}$$

$$\begin{aligned}
 f_4(x) &= \frac{u_{m2}x_5(t)}{k_{s2} + x_5(t)}x_4(t), \\
 f_5(x) &= -\frac{k_2u_{m2}x_5(t)}{k_{s2} + x_5(t)}x_4(t) \\
 &\quad + \frac{Y_bu_{m1}x_3(t)}{k_{s1} + x_3(t)}x_2(t), \\
 B_1 &= (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)^T, \\
 \Lambda &= \begin{pmatrix} \delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},
 \end{aligned}$$

It is easy to deduce that

$$\|f(x)\| \leq \theta_1\|x\| + \theta_2\|x\|^2, \quad (4)$$

where  $\theta_1, \theta_2$  are unknown positive constants.

For the biomass anaerobic digestion system (3), the objective of this paper is to design a sliding mode controller. Under the action of the controller, the state vector of biomass anaerobic digestion system (3) restricted to the specified sliding surface, and the sliding mode motion along the specified sliding surface is asymptotically stable. For the biomass anaerobic digestion system, the quadratic sliding surface can be designed as following.

$$s(t) = Gx(t), \quad (5)$$

where  $G = P^{-1}$ ,  $P$  is a positive definite matrix with appropriate dimension. When the state trajectories of the system (3) is in the sliding mode,  $s(t) = 0$ .

The aim of this paper is that the sliding mode control law can make the state trajectory tend to the designated sliding surface  $s(t) = 0$ , and asymptotically tends to zero along the sliding surface.

The following lemma is used in this paper.

**Lemma 2.1** For a given symmetric matrix  $A \in \mathbb{R}^{n \times n}$ , which is partitioned into blocks

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

where  $A_{11} \in \mathbb{R}^{r \times r}$ ,  $A_{12} \in \mathbb{R}^{r \times (n-r)}$ ,  $A_{21} \in \mathbb{R}^{(n-r) \times r}$ ,  $A_{22} \in \mathbb{R}^{(n-r) \times (n-r)}$ . Then the following three

conditions are equivalent:

- (1)  $A < 0$ ;
- (2)  $A_{11} < 0, A_{22} - A_{21}A_{11}^{-1}A_{12}$ ;
- (3)  $A_{22} < 0, A_{11} - A_{12}A_{22}^{-1}A_{21}$ .

### 3、 Controller design

In order to achieve the purpose of this paper, the adaptive sliding mode reliable controller is designed as follows

$$u(t) = -B^T Gx(t) - \rho(t)B^T \text{sgn}(s(t)), \quad (6)$$

$$\rho(t) = \frac{\mu + \hat{\theta}_1 \|x\| + \hat{\theta}_2 \|x\|^2}{-\|x(t)B^T Gx(t)\|}, \quad (7)$$

the update rates are defined as follows

$$\dot{\hat{\theta}}_1 = l_1 x p^{-1} \|x\|, \quad \hat{\theta}_1(0) > 0, \quad (8)$$

$$\dot{\hat{\theta}}_2 = l_2 x p^{-1} \|x\|^2, \quad \hat{\theta}_2(0) > 0, \quad (9)$$

$$\dot{\hat{d}} = l_3 (1 - d_2) x^T P^{-1} N(x) \rho(t) \text{sgn}(s(t)), \quad \hat{d}(0) > 0, \quad (10)$$

where  $\mu$  is a very small scalar constant.  $l_1, l_2, l_3$  are the update gain.  $\lambda(x)$  is the minimal eigenvalue of matrix  $N(x)N^T(x)$ . It is easy to deduce that  $\lambda(x) \neq 0$  by the definition of  $N(x)$ .

**Theorem 3.1** Considering the biomass anaerobic digestion system (3). The adaptive sliding mode controller is designed as (6), (7), (8), (9) and (10). If there exists matrix  $P > 0$ , such that the following linear matrix inequality is established

$$\begin{pmatrix} \Psi & ' & PG^T \bar{\Lambda} \\ B^T & -I & 0 \\ \bar{\Lambda} GP & 0 & -I \end{pmatrix} < 0, \quad (11)$$

where  $\Psi = -B^T GP - PGB^T$ . Then the closed-loop system (3) is asymptotically stable.

**Proof** Substituting (6) into (3)

$$\dot{x}(t) = f(x) - (N(x)$$

$$+d(I - \Lambda(t))[B^T Gx(t) + \rho(t)sgn(s(t))] \quad (12)$$

Letting the Lyapunov function for the biomass anaerobic digestion system (3)

$$V_1 = x^T P^{-1}x + \frac{1}{l_1}(\hat{\theta}_1 - \theta_1)^2$$

$$+ \frac{1}{l_2}(\hat{\theta}_2 - \theta_2)^2 + \frac{1}{l_3}(\hat{d} - d_1)^2. \quad (13)$$

Differentiating the above Lyapunov function along the biomass anaerobic digestion system (3) yields: \

$$\begin{aligned} \dot{V}_1 &= 2x^T P^{-1}(f(x) \\ &\quad - (N(x) + d(I - \Lambda(t))B)(B^T Gx \\ &\quad + \rho(t)sgn(s(t))) + \frac{2}{l_1}\dot{\hat{\theta}}_1(\hat{\theta}_1 - \theta_1) \\ &\quad + \frac{2}{l_2}\dot{\hat{\theta}}_2(\hat{\theta}_2 - \theta_2) + \frac{2}{l_3}\dot{\hat{d}}(\hat{d} - d_1) \\ &\leq -dx^T (P^{-1}B^T G + GB^T P^{-1})x \\ &\quad + 2\|x^T P^{-1}\|(\theta_1\|x\| + \theta_2\|x\|^2) \\ &\quad + 2x^T P^{-1}xB^T Gx + 2x^T P^{-1}x\rho(t)sgn(s(t)) \\ &\quad + 2dx^T P^{-1}\Lambda(t)B^T Gx \\ &\quad - 2dx^T P^{-1}(I - \Lambda(t))B\rho(t)sgn(s(t)) \\ &\quad + \frac{2}{l_1}\dot{\hat{\theta}}_1(\hat{\theta}_1 - \theta_1) + \frac{2}{l_2}\dot{\hat{\theta}}_2(\hat{\theta}_2 - \theta_2). \quad (14) \end{aligned}$$

By means of the definition of the sliding mode variable  $s(t)$  in (5) and  $\|s(t)\|_1 \geq \|s(t)\|$ , one can obtain

$$\begin{aligned} &-2dx^T P^{-1}(I - \Lambda(t))\rho(t)sgn(s(t)) \\ &= -2dx^T P^{-1} \left[ (1 - \delta_1)|s_1| + \sum_{i=2}^6 |s_i| \right] \\ &\leq -2dx^T P^{-1}[\mu + \hat{\theta}_1 + \hat{\theta}_2 + \hat{d} + \|B^T Gx(t)\|]\|s(t)\|_1 \quad v \leq \end{aligned}$$

$$-2dx^T P^{-1} \left[ \begin{array}{c} \mu + \hat{\theta}_1 + \hat{\theta}_2 + \hat{d} \\ + \|B^T Gx(t)\| \end{array} \right] \|s(t)\| \quad (15)$$

$$2dx^T P^{-1} \Lambda(t) B^T Gx \leq dx^T P^{-1} B B^T P^{-1} x + dx^T G^T \bar{\Lambda}^2 Gx \quad (16)$$

where

$$\bar{\Lambda} = \begin{pmatrix} \delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Substituting (8), (9), (10), (15) and (16) into (14), letting

$$\begin{aligned} \dot{V}_1 &= 2x^T P^{-1} (f(x) \\ &\quad - (N(x) + d(I - \Lambda(t))B)(B^T Gx \\ &\quad + \rho(t) \operatorname{sgn}(s(t))) + \frac{2}{l_1} \dot{\hat{\theta}}_1 (\hat{\theta}_1 - \theta_1) + \frac{2}{l_2} \dot{\hat{\theta}}_2 (\hat{\theta}_2 - \theta_2) + \frac{2}{l_3} \dot{\hat{d}} (\hat{d} - d_1) \\ &\leq -dx^T (P^{-1} B^T G + G B^T P^{-1}) x \\ &\quad + 2 \|x^T P^{-1}\| (\theta_1 \|x\| + \theta_2 \|x\|^2) \\ &\quad + 2x^T P^{-1} x B^T Gx + 2x^T P^{-1} x \rho(t) \operatorname{sgn}(s(t)) \\ &\quad + 2dx^T P^{-1} \Lambda(t) B^T Gx \\ &\quad - 2dx^T P^{-1} (I - \Lambda(t)) B \rho(t) \operatorname{sgn}(s(t)) \\ &\quad + \frac{2}{l_1} \dot{\hat{\theta}}_1 (\hat{\theta}_1 - \theta_1) \\ &\quad + \frac{2}{l_2} \dot{\hat{\theta}}_2 (\hat{\theta}_2 - \theta_2) - \frac{2}{l_3} \dot{\hat{d}} (\hat{d} - d_1) \\ &\leq -dx^T (P^{-1} B^T G + G B^T P^{-1}) x + 2x^T P^{-1} x (B^T Gx + \rho(t) \operatorname{sgn}(s(t))) \\ &\quad - 2dx^T P^{-1} [\mu + \hat{\theta}_1 \|x\| + \hat{\theta}_2 \|x\|^2 - \|x(t) B^T Gx(t)\|] \|s(t)\| \\ &\quad + dx^T P^{-1} B B^T P^{-1} x + dx^T G^T \bar{\Lambda}^2 Gx + 2 \|x^T P^{-1}\| (\theta_1 \|x\| + \theta_2 \|x\|^2) + \frac{2}{l_1} l_1 x p^{-1} \|x\| (\hat{\theta}_1 - \theta_1) \\ &\quad + \frac{2}{l_2} l_2 x p^{-1} \|x\|^2 (\hat{\theta}_2 - \theta_2). \end{aligned}$$

$$\dot{V}_1 \leq dx^T \Psi_1 x,$$

where

$$\Psi_1 = -P^{-1}B^T G - GB^T P^{-1} + P^{-1}BB^T P^{-1} + G^T \bar{\Lambda}^2 G. \quad (17)$$

By means of Lemma 2.1,  $\Psi_1 < 0$  is equivalent to the following linear matrix inequalities

$$\begin{pmatrix} \Psi_2 & P^{-1}B & G^T \bar{\Lambda} \\ B^T P^{-1} & -I & 0 \\ \bar{\Lambda} G & 0 & -I \end{pmatrix} < 0, \quad (18)$$

where  $\Psi_2 = -P^{-1}B^T G - GB^T P^{-1}$ .

The left and right sides of (18) are multiplied by  $diag(P, I, I)$ , then we have

$$\begin{pmatrix} \Psi & B & PG^T \bar{\Lambda} \\ B^T & -I & 0 \\ \bar{\Lambda} GP & 0 & -I \end{pmatrix} < 0.$$

where  $\Psi = -B^T GP - PGB^T$ .

Therefore, we have

$$\dot{V}_1 < 0 \quad (19)$$

Thus system (10) is asymptotically stable, the estimation errors are bounded.

The following conclusion proves that the controller is able to make the system (3) the state trajectory tend to be specified by the sliding surface.

**Theorem 3.2** For the biomass anaerobic digestion system (3), the adaptive sliding mode controller is designed as (6), (7), (8), (9), (10). If there exists matrix,  $P > 0$ , such that the linear matrix inequality (11) is established, then the designed adaptive sliding mode controller is able to guarantee the sliding mode surface  $s(t)=0$  can up to. the proof of the convexity theorem 3.2 is similar to theorem 3.1

## 5. CONCLUSION

In this paper, we focus on the biomass anaerobic digestion process and the biomass anaerobic digestion system with partly uncertain parameters is proposed firstly, which the upper bound of the uncertain parameters is not exactly known. By means of Based on the Lyapunov stability theorem and (LMI),



adaptive sliding mode control schemes of stabilization for the proposed biomass anaerobic digestion system has been addressed and solved in this study.

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