International Journal of Applied Science and Engineering Review

ISSN: 2582-6271

Vol. 3, Issue.2, March-April 2022, page no. 12-31

To cite this article: Adaraniwon Amos Olalekan and Omosigho Donatus Oseretin (2022). DETERIORATING INVENTORY POLICY FOR ITEMS WITH POWER DEMAND AND VARIABLE HOLDING COST CONSIDERING SHORTAGES, International Journal of Applied Science and Engineering Review (IJASER) 3 (2): 12-31

DETERIORATING INVENTORY POLICY FOR ITEMS WITH POWER DEMAND AND VARIABLE HOLDING COST CONSIDERING SHORTAGES

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DOI: http://dx.doi.org/10.52267/IJASER.2022.3202

ABSTRACT

In this paper, An EOQ model for a linear deteriorating inventory policy for items with power demand pattern is presented. Holding cost is assumed to be time-dependent. Shortages are allowed and partially backlogged. The paper aims at minimising the total inventory cost by optimising the scheduling period and optimal ordering quantity. A numerical example is presented to establish the applicability of the model and sensitivity analysis carried out based on the example to know the effect of changes in various parameters in the system.

KEYWORDS: Power pattern, Linear deterioration, Time-dependent, Holding cost, Shortages, Inventory

1. INTRODUCTION

Demand patterns is defined as different ways by which items are withdrawn out of inventory during the scheduling period to supply the need of the customers. The demand patterns is said to be uniform if the demand rate is the same during all the inventory period. One of the advantages of demand pattern is that it allows suiting the demand for more practical situations. Thus, the pattern permits representing the nature of demand when it is uniformly distributed throughout the cycle and also to reflect sales in different phrases of the products life cycle in the market. For example, the demand for inventory increases overtime during the growing period and a decrease in the decline phrase. In this work, demand is assumed to follows power pattern.

Many papers on inventory policy consider that the demand follows a power pattern.[9] proposed an order-level inventory policy for deteriorating items with power demand pattern.[12] studied an inventory model that has a variable rate of deterioration and the demand follows a power pattern. Also, [11] investigated a deteriorating inventory model for an item with power demand pattern with shortages. [13,14] extended Lee's model to a general class with backlogging rate which is time proportional. Other interesting works can be found in [10, 17, 16, 15, 7, 19, 18, 6]

In the management of inventory system, deterioration plays a very important role. Items kept for future use normally lose part of their value steadily with time, which is otherwise refers to as deterioration. Deterioration is defined as decrease, decline, damage, evaporation in the degree of excellence of a product over a period of time until the items become unusable. It may be as a result of the item having extended it's fixed life span, or because the storage facilities are insufficient or unsatisfactory and also it may be as a result of careless handling of the items in the storehouse/warehouse. Examples of such items include medicine, films, milk, meats fruits, vegetable etc. In analysing inventory system, deterioration must be taken into consideration to avoid shortages and to keep the system at optimum. The study of inventory that deteriorates over a period actually started with [20] who proposed a fashion items that deteriorate at the end of the storage period.

2. Assumptions and notation

The inventory mathematical model for this work is developed based on the following assumptions and notation.

Notation:

- T :Length of the inventory cycle.
- t_1:Time at which the inventory depleted to 0.
- $Q_1(t)$: Positive inventory level at time t.
- $Q_2(t)$: Negative inventory level at time t.
- θ: Deteriorating rate $(0 < \theta \leq 1)$
- P: Ordering quantity (units)
- M:Maximum inventory level during the cycle
- N: Maximum inventory level during negative inventory period
- d: Average demand per scheduling period per units per time
- γ: Backlogging rate. $(0 \leq \gamma \leq 1)$
- n :Demand pattern index, (n must be greater than 0)
- A :Ordering cost ($ per order)
- h :Holding cost per unit ($ per /time/unit)
- Z :Purchasing cost per unit ($ per unit).
- K :Cost per shortage unit ($ per unit).
- S :Cost per lost sale unit ($ per unit).

- HC :Holding cost per/time/unit.
- SC :Shortage cost per/time/unit
- LSC :Lost sale cost per/time/unit.
- TC :Total cost of the inventory policy per/time/unit.

Assumptions: .

1. Demand is power demand pattern.
2. Shortages is permitted and partially backlogged
3. Deterioration rate of item is a linear function of time
4. Holding cost is time- dependent and taken as $h(t) = h + \beta t$, where $h > 0, \beta > 0$
5. Lead time is negligible.
6. Replenishment is instantaneous.
7. Demand $D(t)$ varies with time and taken as $D(t) = \frac{dt^{\frac{1}{n}}}{nT^{\frac{1}{n}-1}}$,where r is the average demand and the power index is n, $0 < n < \infty$ and $0 \leq t \leq T$.

   The rate of demand at any given time t is $D'(t) = \frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}}$.

3. Mathematical Formulation

Let $q_1(t)$ be the stock level at time t which ranges between $0 \leq t \leq T$.At the onset of the inventory cycle, the maximum inventory level $Q_1(0) = M$ reduces as a result of demand and the process of deterioration also set in for the items.At the interval $t = t_1$, the inventory system get down to zero level.Thereafter, at the interval $[t_1, T]$, shortages occurs in the system and they are backlogged to the end of the cycle. At the interval $t = T$, the system reach a level N.

The inventory level $Q_1(t)$ and $Q_2(t)$ during the cycle period is describes in the Figure 1. Based on the above assumptions, the differential equations that represent stock level is given as:

ISSN: 2582-6271

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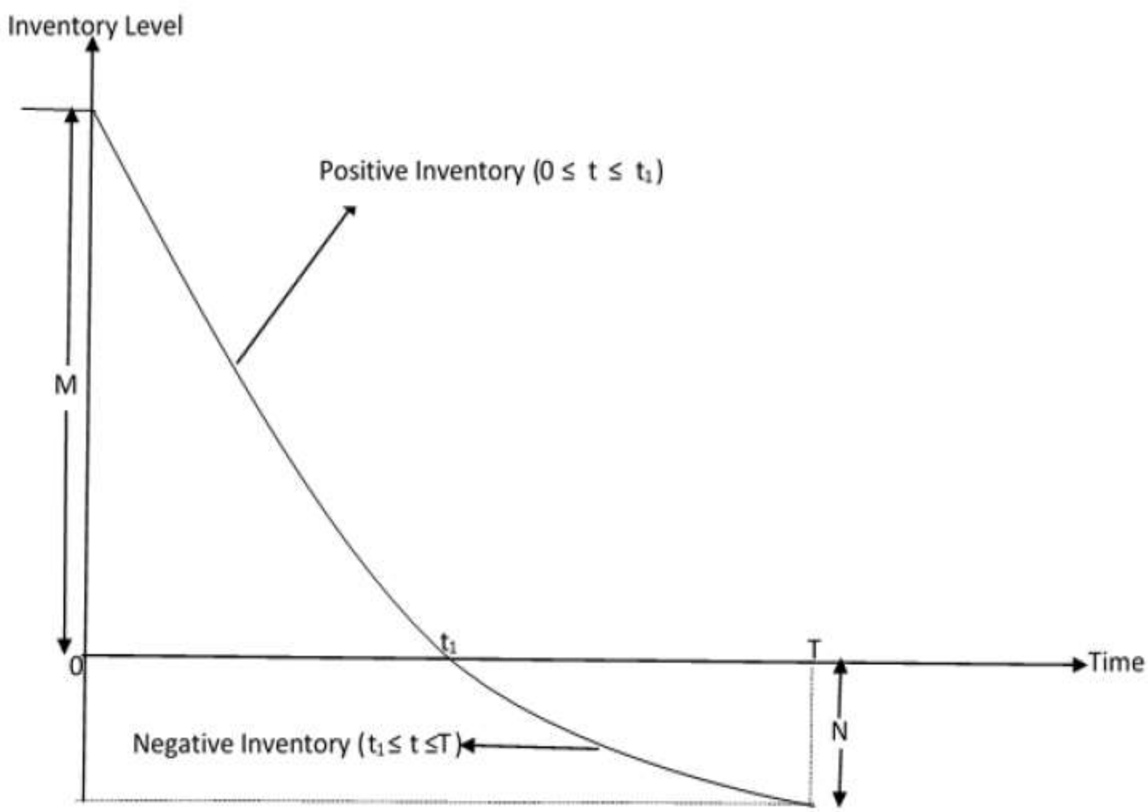

Fig. 1: Graphical Representation of Inventory Model

$$\frac{dQ_1(t)}{dt} + \theta t Q(t) = \frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}-1}} \quad 0 \le \ t \le t_1 \qquad (1)$$

with the boundary conditions $Q_1(0) = M, Q_1(t_1) = 0 \ \ and \ \ Q_2(t_1) = 0$

using the integrating factor $e^{\int pdt} = e^{\frac{\theta t^2}{2}}$

$$\frac{Q_1(t)e^{\frac{\theta t^2}{2}}}{dt} + \theta t Q_1(t) e^{\frac{\theta t^2}{2}} = -\frac{d}{nT^{\frac{1}{n}-1}}\left[t^{\frac{1}{n}-1} e^{\frac{\theta t^2}{2}}\right]$$

$$Q_1' e^{\frac{\theta t^2}{2}} = -\frac{d}{nT^{\frac{1}{n}-1}}\left[\int \ t^{\frac{1}{n}-1} e^{\frac{\theta t^2}{2}}\right] dt$$

Since θ is small,0 $\le \theta \le 1$, taking the first three terms of the power series,we have:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots\ldots\ldots..$$

$$e^{\frac{\theta t^2}{2}} = 1 + \frac{\theta t^2}{2} + \frac{(\theta t^2)^2}{8} + \frac{(\theta t^2)^3}{24} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$Q_1'(t)e^{\frac{\theta t^2}{2}} = -\frac{d}{nT^{\frac{1}{n}-1}}\left[\int t^{\frac{1}{n}-1}\left(1 + \frac{\theta t^2}{2} + \frac{\theta^2 t^4}{8}\right)\right] dt$$

$$Q_1(t)e^{\frac{\theta t^2}{2}} = -\frac{d}{nT^{\frac{1}{n}-1}}\left[nt^{\frac{1}{n}} + \frac{n\theta t^{\frac{1}{n}+2}}{2(2n+1)} + \frac{n\theta^2 t^{\frac{1}{n}+4}}{8(4n+1)}\right] + C$$

Using the boundary condition $Q_1(0) = M$

$M = C$, therefore:

$$Q_1(t)e^{\frac{\theta t^2}{2}} = M - \frac{d}{nT^{\frac{1}{n}-1}}\left[nt^{\frac{1}{n}} + \frac{n\theta t^{\frac{1}{n}+2}}{2(2n+1)} + \frac{n\theta^2 t^{\frac{1}{n}+4}}{8(4n+1)}\right]$$

$$Q_1(t) = Me^{-\frac{\theta t^2}{2}} - \frac{de^{-\frac{\theta t^2}{2}}}{T^{\frac{1}{n}-1}}\left[t^{\frac{1}{n}} + \frac{\theta t^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t^{\frac{1}{n}+4}}{8(4n+1)}\right] \quad (2)$$

During the negative inventory, the policy is represented by the equation:

$$\frac{Q_2(t)}{dt} = -\gamma D(t) \quad t_1 \leq \ t \leq T \quad (3)$$

$$Q_2'(t) = -\frac{\gamma d}{nT^{\frac{1}{n}-1}}\int t^{\frac{1}{n}-1} dt$$

$$Q_2(t) = -\frac{\gamma d}{T^{\frac{1}{n}-1}}\left(t^{\frac{1}{n}}\right) + C$$

Using the boundary condition $Q_2(t_1) = 0$

$$0 = -\frac{\gamma d}{T^{\frac{1}{n}-1}} t_1^{\frac{1}{n}} + C$$

$$C = \frac{\gamma d}{T^{\frac{1}{n}-1}} t_1^{\frac{1}{n}}$$

Therefore:

$$Q_2(t) = \frac{\gamma d}{T^{\frac{1}{n}-1}}\left[t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right] \quad (4)$$

Q_1 is a continuously decreasing function in the time interval $[0, T]$, therefore the initial net stock level at this interval is obtained by substituting the boundary condition $Q_{t_1} = 0$ into the Equation 3, we have

$$0 = Me^{-\frac{\theta t_1^2}{2}} - \frac{de^{-\frac{\theta t_1^2}{2}}}{T^{\frac{1}{n}-1}}\left[t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\right]$$

$$Me^{-\frac{\theta t_1^2}{2}} = \frac{de^{-\frac{\theta t_1^2}{2}}}{T^{\frac{1}{n}-1}}\left[t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\right]$$

$$M = \frac{d}{T^{\frac{1}{n}-1}}\left[t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\right] \qquad (5)$$

The maximum negative inventory per units is given as:

$$N = -Q_2(T)$$

$$N = \frac{\gamma d}{T^{\frac{1}{n}-1}}\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) \qquad (6)$$

The order size during the entire period $[0, T]$ is given as:

$$P = M + N$$

$$P = \frac{\gamma d}{T^{\frac{1}{n}-1}}\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{d}{T^{\frac{1}{n}-1}}\left[t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\right]$$

$$P = \frac{d}{T^{\frac{1}{n}-1}}\left[\gamma T^{\frac{1}{n}} - \gamma t_1^{\frac{1}{n}} + t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\right] \qquad (7)$$

The cost of holding inventory occurs at the interval $[0, t_1]$ only, Hence the holding cost during this interval $[0, t_1]$ is obtained as follows:

$$HC = \int_0^{t_1} h(t)Q_1(t)dt$$

$$HC = \int_0^{t_1} (h + \beta t)Q_1 dt$$

$$HC = \int_0^{t_1} (h + \beta t)\left[t_1^{\frac{1}{n}} - t^{\frac{1}{n}} - \frac{\theta t^2 t_1^{\frac{1}{n}}}{2} + \frac{\theta^2 t^4 t_1^{\frac{1}{n}}}{8} + \frac{\theta n t^{\frac{1}{n}+2}}{2n+1} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} - \frac{\theta^2 t^2 t_1^{\frac{1}{n}+2}}{4(2n+1)} - \frac{\theta^2 t^{\frac{1}{n}+4}}{8} + \frac{\theta^2 t^{\frac{1}{n}+4}}{4(2n+1)} - \frac{\theta^2 t^{\frac{1}{n}+4}}{8(4n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\right]$$

Upon expansion and some simplifications,we have:

$$HC = \frac{d}{T^{\frac{1}{n}-1}}\left[ht_1^{\frac{1}{n}+1} - \frac{hnt_1^{\frac{1}{n}+1}}{n+1} - \frac{h\theta t_1^{\frac{1}{n}+3}}{6} + \frac{h\theta^2 t_1^{\frac{1}{n}+5}}{40} + \frac{h\theta n^2 t_1^{\frac{1}{n}+3}}{(2n+1)(3n+1)}\right.$$

$$+\frac{h\theta t_1^{\frac{1}{n}+3}}{2(2n+1)}-\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{12(2n+1)}-\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{8(5n+1)}+\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{4(5n+1)(2n+1)}$$
$$-\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{8(5n+1)(4n+1)}+\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)}+\frac{\beta t_1^{\frac{1}{n}+2}}{2}-\frac{\beta n t_1^{\frac{1}{n}+2}}{2n+1}-\frac{\beta\theta t_1^{\frac{1}{n}+4}}{8}$$
$$+\frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{48}+\frac{\beta\theta^2 n^2 t_1^{\frac{1}{n}+4}}{(2n+1)(4n+1)}+\frac{\beta\theta t_1^{\frac{1}{n}+4}}{4(2n+1)}-\frac{\beta\theta^2 t_1^{\frac{1}{n}+7}}{16(2n+1)}-\frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{8(6n+1)}$$
$$+\frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{8(6n+1)}+\frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{4(2n+1)(6n+1)}-\frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{8(4n+1)(6n+1)}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{16(4n+1)}\Bigg] \quad (8)$$

Purchase cost is obtained thus:

$$PC = Z\left(M+\int_{t_1}^{T}\gamma D(t)\right)dt$$
$$PC = \frac{Zd}{T^{\frac{1}{n}-1}}\left(t_1^{\frac{1}{n}}+\frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)}+\frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}+\frac{\gamma}{nT^{\frac{1}{n}-1}}\left(\int_{t_1}^{T} t^{\frac{1}{n}-1}dt\right)\right)$$
$$PC = \frac{Zd}{T^{\frac{1}{n}-1}}\left[t_1^{\frac{1}{n}}+\frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)}+\frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}+\gamma(T^{\frac{1}{n}}-t_1^{\frac{1}{n}})\right] \quad (9)$$

Shortages due to stock out is accumulated in the policy during the interval $[t_1, T]$.

The policy attains optimum level of shortage at $(t = T)$, hence the overall shortage cost at this period of time is obtained thus:

$$SC = K\int_{t_1}^{T} -Q_2(t)dt$$
$$SC = K\int_{t_1}^{T} -\frac{\gamma d}{T^{\frac{1}{n}-1}}(t_1^{\frac{1}{n}}-t^{\frac{1}{n}})dt$$
$$SC = \frac{Kd\gamma}{T^{\frac{1}{n}-1}}\left[Tt_1^{\frac{1}{n}}-\frac{nT^{\frac{1}{n}+1}}{n+1}-\frac{t_1^{\frac{1}{n}+1}}{n+1}\right] \quad (10)$$

As a result of stock out during (t_1, T), Shortage is accumulated, but not all customers are willing to wait for the next lot size to arrive. Hence this result in some loss of sale which account to the loss in profits.

Lost sale cost is calculated as follows:

$$LSC = S\int_{t1}^{T}(1-\gamma)D(t)dt$$

$$LSC = \frac{Sd(1-\gamma)}{T^{\frac{1}{n}-1}}\left[T^{\frac{1}{n}} - t^{\frac{1}{n}}\right] \tag{11}$$

The total cost for the inventory system consist of the following cost components:

TC = Ordering cost + Holding cost + Purchase cost + Shortage cost + Lost sale cost /T

$$\begin{aligned} TC = \frac{d}{T^{\frac{1}{n}-1}}\Bigg[& ht_1^{\frac{1}{n}+1} - \frac{hnt_1^{\frac{1}{n}+1}}{n+1} - \frac{h\theta t_1^{\frac{1}{n}+3}}{6} + \frac{h\theta^2 t_1^{\frac{1}{n}+5}}{40} + \frac{h\theta n^2 t_1^{\frac{1}{n}+3}}{(2n+1)(3n+1)} \\ & + \frac{h\theta t_1^{\frac{1}{n}+3}}{2(2n+1)} - \frac{h\theta^2 t_1^{\frac{1}{n}+5}}{12(2n+1)} - \frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{8(5n+1)} + \frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{4(5n+1)(2n+1)} \\ & - \frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{8(5n+1)(4n+1)} + \frac{h\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)} + \frac{\beta t_1^{\frac{1}{n}+2}}{2} - \frac{\beta n t_1^{\frac{1}{n}+2}}{2n+1} - \frac{\beta\theta t_1^{\frac{1}{n}+4}}{8} \\ & + \frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{48} + \frac{\beta\theta^2 n^2 t_1^{\frac{1}{n}+4}}{(2n+1)(4n+1)} + \frac{\beta\theta t_1^{\frac{1}{n}+4}}{4(2n+1)} - \frac{\beta\theta^2 t_1^{\frac{1}{n}+7}}{16(2n+1)} - \frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{8(6n+1)} \\ & + \frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{8(6n+1)} + \frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{4(2n+1)(6n+1)} - \frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{8(4n+1)(6n+1)} + \frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{16(4n+1)}\Bigg] \\ & + \frac{Zd}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}} + \frac{\theta t_1^{\frac{1}{n}+2}}{2(2n+1)} + \frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)} + \gamma(T^{\frac{1}{n}} - t_1^{\frac{1}{n}})\right] \\ & + \frac{Kd\gamma}{T^{\frac{1}{n}}}\left[Tt_1^{\frac{1}{n}} - \frac{nT^{\frac{1}{n}+1}}{n+1} - \frac{t_1^{\frac{1}{n}+1}}{n+1}\right] \\ & + \frac{Sd(1-\gamma)}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} - t^{\frac{1}{n}}\right] + \frac{A}{T} \end{aligned} \tag{12}$$

4. Solution Method

We present an approach to determine the inventory policy that minimizes the total inventory cost per unit time in this section. From (12), we find the first partial derivative of $TC(T, t_1)$ with respect to the decision variables T and t_1:

We obtain:

$$\frac{\partial TC(t_1, T)}{\partial t_1} \quad and \quad \frac{\partial TC(t_1, T)}{\partial T}$$

To minimize the total cost $TC(t_1, T)$ per unit time, the optimal value of T *and* t_1 can be obtained by solving the Equations:

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$$\frac{\partial TC(t_1,T)}{\partial t_1}=0 \quad and \quad \frac{\partial TC(t_1,T)}{\partial T}=0 \tag{13}$$

Provided that equation (12) satisfies the following conditions:

$$\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)>0 \quad and \quad \left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)>0 \tag{14}$$

$$\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1 \partial T}\right)^2>0 \tag{15}$$

$$\begin{aligned}\frac{\partial TC(t_1,T)}{\partial t_1}=\frac{d}{T^{\frac{1}{n}}}\Bigg[&\frac{h(n+1)t^{\frac{1}{n}}}{n}-ht_1^{\frac{1}{n}}-\frac{h\theta(3n+1)t_1^{\frac{1}{n}+2}}{6n}+\frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{40n}\\&+\frac{h\theta n t_1^{\frac{1}{n}+2}}{2n+1}+\frac{h\theta(3n+1)t_1^{\frac{1}{n}+2}}{2n(2n+1)}-\frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{2n(2n+1)}-\frac{h\theta^2 t_1^{\frac{1}{n}+4}}{8}\\&+\frac{h\theta^2 t_1^{\frac{1}{n}+4}}{4(2n+1)}-\frac{h\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}+\frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{8n(4n+1)}+\frac{\beta(2n+1)t_1^{\frac{1}{n}+1}}{2n}-\beta t_1^{\frac{1}{n}+1}\\&-\frac{\beta\theta(4n+1)t_1^{\frac{1}{n}+3}}{8n}+\frac{\beta\theta^2(6n+1)t_1^{\frac{1}{n}+5}}{48n}+\frac{\beta\theta^2 n t_1^{\frac{1}{n}+3}}{(2n+1)}+\frac{\beta\theta(4n+1)t_1^{\frac{1}{n}+3}}{4n(2n+1)}\\&-\frac{\beta\theta^2(7n+1)t_1^{\frac{1}{n}+6}}{16n(2n+1)}-\frac{\beta\theta^2 t_1^{\frac{1}{n}+5}}{8}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+5}}{4(2n+1)}-\frac{\beta\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)}+\frac{\beta\theta^2(6n+1)t_1^{\frac{1}{n}+5}}{16n(4n+1)}\Bigg]\\&\frac{Zd}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}-1}}{n}+\frac{\theta t_1^{\frac{1}{n}+1}}{2n}+\frac{\theta^2 t_1^{\frac{1}{n}+3}}{8n}-\frac{\gamma t_1^{\frac{1}{n}-1}}{n}\right]-\frac{K\gamma d}{T^{\frac{1}{n}}}\left[\frac{Tt_1^{\frac{1}{n}-1}}{n}-\frac{t_1^{\frac{1}{n}}}{n}\right]\\&-\frac{Sd(1-\gamma)}{T^{\frac{1}{n}}}\left(\frac{t_1^{\frac{1}{n}-1}}{n}\right)+\frac{A}{T}=0\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial TC(t_1,T)}{\partial T}=\frac{-A}{T^2}-\frac{d}{nT^{\frac{1}{n}+1}}\Bigg[&ht^{\frac{1}{n}+1}-\frac{hnt_1^{\frac{1}{n}+1}}{n+1}+\frac{h\theta t_1^{\frac{1}{n}+3}}{6}+\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{40}+\frac{hn^2\theta t_1^{\frac{1}{n}+3}}{(2n+1)(3n+1)}\\&+\frac{h\theta t_1^{\frac{1}{n}+3}}{4n+2}-\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{4n+2}-\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{4n+8}+\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{4(2n+1)(5n+1)}-\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)(5n+1)}\\&+\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)}+\frac{\beta t_1^{\frac{1}{n}+2}}{2}-\frac{\beta n t_1^{\frac{1}{n}+2}}{2n+1}-\frac{\beta\theta t_1^{\frac{1}{n}+4}}{8}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{48}+\frac{\beta\theta^2 n^2 t_1^{\frac{1}{n}+4}}{(2n+1)(4n+1)}\end{aligned}$$

$$+\frac{\beta\theta t_1^{\frac{1}{n}+4}}{8n+4}-\frac{\beta\theta^2 t_1^{\frac{1}{n}+7}}{32n+16}-\frac{\beta\theta^2 n t_1^{\frac{1}{n}+6}}{48n+8}+\frac{\beta n\theta^2 t_1^{\frac{1}{n}+2}}{4(2n+1)(6n+1)}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{64n+16}$$
$$-\frac{\beta\theta^2 n t_1^{\frac{1}{n}+2}}{8(4n+1)(6n+1)}\Bigg]-\frac{ZD}{nT^{\frac{1}{n}+1}}\left[t_1^{\frac{1}{n}}+\frac{\theta t_1^{\frac{1}{n}+2}}{4n+2}+\frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}+\gamma T^{\frac{1}{n}}-\gamma t^{\frac{1}{n}}\right]$$
$$+\frac{Zd\gamma}{nT}+\frac{K\gamma d}{nT^{\frac{1}{n}+1}}\left[Tt_1^{\frac{1}{n}}-\frac{nT^{\frac{1}{n}+1}}{n+1}-\frac{t_1^{\frac{1}{n}+1}}{n+1}\right]-\frac{Kd\gamma(t_1^{\frac{1}{1}}-T^{\frac{1}{n}})}{T^{\frac{1}{n}}}$$
$$+\frac{Sd(1-\gamma)}{nT}-\frac{Sd(1-\gamma)(T^{\frac{1}{n}}-t_1^{\frac{1}{n}})}{nT^{\frac{1}{n}+1}} \qquad (17)$$

$$\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}=\frac{d}{T^{\frac{1}{n}}}\Bigg[t_1^{\frac{1}{n}-1}h(\frac{1}{n}+1)^2+\frac{hn^2\theta(\frac{1}{n}+3)^2 t_1^{\frac{1}{n}+1}}{(2n+1)(3n+1)}+\frac{hn\theta^2(\frac{1}{n}+5)^2 t_1^{\frac{1}{n}+3}}{4(2n+1)(5n+1)}+ht_1^{\frac{1}{n}-1}$$
$$-\frac{hn\theta^2(\frac{1}{n}+5)^2 t_1^{\frac{1}{n}+3}}{8(4n+1)(5n+1)}-\frac{h\theta(\frac{1}{n}+3)^2 t_1^{\frac{1}{n}+1}}{6}+\frac{h\theta^2(\frac{1}{n}+5)^2 t_1^{\frac{1}{n}+3}}{40}-\frac{h\theta(\frac{1}{n}+3)t_1^{\frac{1}{n}+1}}{(4n+2)}$$
$$+\frac{h\theta^2(\frac{1}{n}+5)t_1^{\frac{1}{n}+3}}{(4n+2)}-\frac{h\theta^2(\frac{1}{n}+5)t_1^{\frac{1}{n}+3}}{8(4n+1)}+\frac{h\theta(\frac{1}{n}+3)t_1^{\frac{1}{n}+1}}{6}-\frac{h\theta^2(\frac{1}{n}+5)t_1^{\frac{1}{n}+3}}{40}$$
$$-\frac{hn\theta^2(\frac{1}{n}+5)t_1^{\frac{1}{n}+3}}{8(5n+1)}+\frac{h\theta^2(\frac{1}{n}+5)^2 t_1^{\frac{1}{n}+3}}{(32n+8)}+\frac{h\theta(\frac{1}{n}+3)^2 t_1^{\frac{1}{n}+1}}{(4n+2)}-\frac{h\theta^2(\frac{1}{n}+5)^2 t_1^{\frac{1}{n}+3}}{(4n+2)}$$
$$-\frac{hn(\frac{1}{n}+1)^2 t_1^{\frac{1}{n}-1}}{(n+1)}-\frac{h\theta^2 t_1^{\frac{1}{n}+3}}{4(2n+1)}+\frac{h\theta^2 t_1^{\frac{1}{n}+3}}{8(4n+1)}-\frac{hn\theta t_1^{\frac{1}{n}+1}}{(2n+1)}+\frac{h\theta^2 t_1^{\frac{1}{n}+3}}{8}$$
$$+ht_1^{\frac{1}{n}-1}(\frac{1}{n}+1)+\frac{\beta\theta^2 n^2(\frac{1}{n}+4)^2 t_1^{\frac{1}{n}+2}}{(4n+1)(2n+1)}+\frac{\beta\theta^2 n(\frac{1}{n}+6)^2 t_1^{\frac{1}{n}+4}}{4(6n+1)(2n+1)}+\beta t_1^{\frac{1}{n}}$$
$$-\frac{\beta\theta(\frac{1}{n}+4)^2 t_1^{\frac{1}{n}+2}}{8}+\frac{\beta\theta^2(\frac{1}{n}+6)^2 t_1^{\frac{1}{n}+4}}{48}-\frac{\beta\theta(\frac{1}{n}+4)^2 t_1^{\frac{1}{n}+2}}{(8n+4)}+\frac{\beta\theta^2(\frac{1}{n}+7)t_1^{\frac{1}{n}+5}}{(32n+16)}$$
$$-\frac{\beta\theta^2(\frac{1}{n}+6)t_1^{\frac{1}{n}+4}}{(64n+16)}-\frac{\beta\theta^2 n(\frac{1}{n}+6)^2 t_1^{\frac{1}{n}+4}}{8(4n+1)(6n+1)}-\frac{\beta\theta^2(\frac{1}{n}+6)t_1^{\frac{1}{n}+4}}{48}-\frac{\beta\theta^2 n(\frac{1}{n}+6)^2 t_1^{\frac{1}{n}+4}}{48n+8}$$
$$+\frac{\beta\theta(\frac{1}{n}+4)^2 t_1^{\frac{1}{n}+2}}{8n+4}+\frac{\beta\theta^2(\frac{1}{n}+6)^2 t_1^{\frac{1}{n}+4}}{64n+16}+\frac{\beta\theta^2 n(\frac{1}{n}+6)t_1^{\frac{1}{n}+4}}{(48n+8)}-\frac{\beta\theta^2(\frac{1}{n}+7)^2 t_1^{\frac{1}{n}+5}}{(32n+16)}$$
$$-\frac{\beta n(\frac{1}{n}+2)^2 t_1^{\frac{1}{n}}}{(2n+1)}-\frac{\beta(\frac{1}{n}+2)t_1^{\frac{1}{n}}}{2}+\frac{\beta(\frac{1}{n}+2)^2 t_1^{\frac{1}{n}}}{2}+\frac{\beta\theta(\frac{1}{n}+4)t_1^{\frac{1}{n}+2}}{8}-\frac{\beta\theta^2 n t_1^{\frac{1}{n}+2}}{(2n+1)}$$

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$$-\frac{\beta\theta^2 t_1^{\frac{1}{n}+4}}{4(2n+1)}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}\Bigg]+\frac{Zd}{T^{\frac{1}{n}}}\Bigg[\frac{t_1^{\frac{1}{n}-2}}{n^2}+\frac{t_1^{\frac{1}{n}-2}}{n}+\frac{\theta t_1^{\frac{1}{n}}(\frac{1}{n}+2)^2}{4n+2}+\frac{\theta t_1^{\frac{1}{n}}(\frac{1}{n}+2)}{4n+2}$$
$$+\frac{\theta^2 t_1^{\frac{1}{n}+2}(\frac{1}{n}+4)^2}{32n+8}-\frac{\theta^2 t_1^{\frac{1}{n}+2}}{8n}-\frac{\gamma t_1^{\frac{1}{n}-2}}{n^2}+\frac{\gamma t_1^{\frac{1}{n}-2}}{n}\Bigg]-\frac{K\gamma d}{T^{\frac{1}{n}}}\Bigg[\frac{Tt_1^{\frac{1}{n}-2}}{n^2}-\frac{Tt_1^{\frac{1}{n}-2}}{n}$$
$$-\frac{(\frac{1}{n}+1)^2 t_1^{\frac{1}{n}-1}}{n+1}+\frac{t_1^{\frac{1}{n}-1}}{n}\Bigg]-\frac{Sd(1-\gamma)t_1^{\frac{1}{n}-2}}{n^2T^{\frac{1}{n}}}+\frac{Sd(1-\gamma)t_1^{\frac{1}{n}-2}}{nT^{\frac{1}{n}}} \tag{18}$$

$$\frac{\partial^2 TC(t_1,T)}{\partial T^2}=\frac{2A}{T^3}+\frac{d}{n^2T^{\frac{1}{n}+2}}\Bigg[ht^{\frac{1}{n}+1}-\frac{hnt_1^{\frac{1}{n}+1}}{n+1}+\frac{h\theta t_1^{\frac{1}{n}+3}}{6}+\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{40}+\frac{hn^2\theta t_1^{\frac{1}{n}+3}}{(2n+1)(3n+1)}$$
$$+\frac{h\theta t_1^{\frac{1}{n}+3}}{4n+2}-\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{4n+2}-\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{4n+8}+\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{4(2n+1)(5n+1)}-\frac{hn\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)(5n+1)}$$
$$+\frac{h\theta^2 t_1^{\frac{1}{n}+5}}{8(4n+1)}+\frac{\beta t_1^{\frac{1}{n}+2}}{2}-\frac{\beta nt_1^{\frac{1}{n}+2}}{2n+1}-\frac{\beta\theta t_1^{\frac{1}{n}+4}}{8}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{48}+\frac{\beta\theta^2 n^2 t_1^{\frac{1}{n}+4}}{(2n+1)(4n+1)}$$
$$+\frac{\beta\theta t_1^{\frac{1}{n}+4}}{8n+4}-\frac{\beta\theta^2 t_1^{\frac{1}{n}+7}}{32n+16}-\frac{\beta\theta^2 nt_1^{\frac{1}{n}+6}}{48n+8}+\frac{\beta n\theta^2 t_1^{\frac{1}{n}+2}}{4(2n+1)(6n+1)}+\frac{\beta\theta^2 t_1^{\frac{1}{n}+6}}{64n+16}$$
$$-\frac{\beta\theta^2 nt_1^{\frac{1}{n}+2}}{8(4n+1)(6n+1)}\Bigg]+\frac{ZD}{n^2T^{\frac{1}{n}+2}}\Bigg[t_1^{\frac{1}{n}}+\frac{\theta t_1^{\frac{1}{n}+2}}{4n+2}+\frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}+\gamma T^{\frac{1}{n}}-\gamma t^{\frac{1}{n}}\Bigg]$$
$$+\frac{ZD}{nT^{\frac{1}{n}+2}}\Bigg[t_1^{\frac{1}{n}}+\frac{\theta t_1^{\frac{1}{n}+2}}{4n+2}+\frac{\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)}+\gamma T^{\frac{1}{n}}-\gamma t^{\frac{1}{n}}\Bigg]-\frac{Zd\gamma}{n^2T^2}-\frac{Zd\gamma}{nT^2}$$
$$-\frac{K\gamma d}{n^2T^{\frac{1}{n}+2}}\Bigg[Tt_1^{\frac{1}{n}}-\frac{nT^{\frac{1}{n}+1}}{n+1}-\frac{t_1^{\frac{1}{n}+1}}{n+1}\Bigg]+\frac{2Kd\gamma(t_1^{\frac{1}{n}}-T^{\frac{1}{n}})}{nT^{\frac{1}{n}+1}}+\frac{Sd(1-\gamma)}{nT^2}$$
$$-\frac{K\gamma d}{nT^{\frac{1}{n}+2}}\Bigg[Tt_1^{\frac{1}{n}}-\frac{nT^{\frac{1}{n}+1}}{n+1}-\frac{t_1^{\frac{1}{n}+1}}{n+1}\Bigg]-\frac{K\gamma d}{T^{\frac{1}{n}}}\left(\frac{-nT^{\frac{1}{n}+1}(\frac{1}{n}+1)^2}{T^2(n+1)}+T^{\frac{1}{n}-1}\right)$$
$$+\frac{Sd(1-\gamma)(T^{\frac{1}{n}}-t_1^{\frac{1}{n}})}{n^2T^{\frac{1}{n}+2}}++\frac{Sd(1-\gamma)(T^{\frac{1}{n}}-t_1^{\frac{1}{n}})}{nT^{\frac{1}{n}+2}}-\frac{Sd(1-\gamma)}{n^2T^2} \tag{19}$$

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$$\frac{\partial^2 TC(t_1,T)}{\partial T \partial t_1} = \frac{-d}{nT^{\frac{1}{n}+1}}\left[\frac{h(1+n)t^{\frac{1}{n}}}{n} - ht_1^{\frac{1}{n}} - \frac{h\theta(\frac{1}{n}+3)t_1^{\frac{1}{n}+2}}{6} + \frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{40n}\right.$$
$$+\frac{hn\theta(3n+1)t_1^{\frac{1}{n}+2}}{2n+1} + \frac{h\theta(3n+1)t_1^{\frac{1}{n}+2}}{n(4n+2)} - \frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{n(4n+2)}$$
$$-\frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{40n+8} + \frac{h\theta^2 t_1^{\frac{1}{n}+4}}{4(2n+1)} - \frac{h\theta^2 t_1^{\frac{1}{n}+4}}{8(4n+1)} + \frac{h\theta^2(5n+1)t_1^{\frac{1}{n}+4}}{8n(4n+1)}$$
$$+\frac{\beta(2n+1)t_1^{\frac{1}{n}+1}}{2n} - \beta(2n+1)t_1^{\frac{1}{n}+1} + \frac{\beta\theta^2(6n+1)t_1^{\frac{1}{n}+5}}{48n}$$
$$+\frac{\theta^2 n\beta(4n+1)t_1^{\frac{1}{n}+3}}{(4n+1)(2n+1)} + \frac{\theta\beta(4n+1)t_1^{\frac{1}{n}+3}}{n(8n+4)} - \frac{\theta^2\beta(7n+1)t_1^{\frac{1}{n}+6}}{16n(2n+1)}$$
$$-\frac{\theta^2\beta(6n+1)t_1^{\frac{1}{n}+5}}{48n+8} + \frac{\theta^2\beta t_1^{\frac{1}{n}+5}}{4(2n+1)} - \frac{\theta^2\beta t_1^{\frac{1}{n}+5}}{8(4n+1)} + \frac{\theta^2\beta(6n+1)t_1^{\frac{1}{n}+5}}{16n(4n+1)}$$
$$\left.-\frac{\beta\theta(4n+1)t_1^{\frac{1}{n}+3}}{8n}\right] - \frac{Zd}{nT^{\frac{1}{n}+1}}\left[\frac{t_1^{\frac{1}{n}-1}}{n} + \frac{\theta t_1^{\frac{1}{n}+1}}{2n} + \frac{\theta^2 t_1^{\frac{1}{n}+3}}{8n} - \frac{\gamma t_1^{\frac{1}{n}+1}}{n}\right]$$
$$+\frac{K\gamma d}{nT^{\frac{1}{n}+1}}\left[\frac{Tt_1^{\frac{1}{n}-1}}{n} - \frac{t_1^{\frac{1}{n}}}{n}\right] - \frac{Kd\gamma t_1^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} + \frac{Sd(1-\gamma)t_1^{\frac{1}{n}-1}}{n^2T^{\frac{1}{n}+1}} \quad (20)$$

Equation (16) and (17) are highly non linear, the values of t_1 and T are solve for the optimal values in order to find minimum total inventory cost per unit time. Maple software 2018 and Excel was utilized to obtained the values of the decision variables.

5. Numerical Example

Here, we give an example to illustrate the results derived from the linear deteriorating inventory policy for items with power demand pattern and variable holding coat considering shortages.

Example.

The following parametric values are considered for the inventory policy in their respective units $A = 500$, $d = 100$, $h = 0.4$ units, $\beta = 15$, $K = \$10$ per units,$S = \$8$ per units, $Z = \$12$ per units, $\theta = 0.8$, $n = 0.5$, $\gamma = 0.6$

Solving equations 13 and 14, The optimal value of $T = 1.670$ and $t_1 = 0.593$

Using these values of t_1 and T,the second derivatives can be found. Hence $\frac{\partial^2 TC(t_1,T)}{\partial t_1^2} =$

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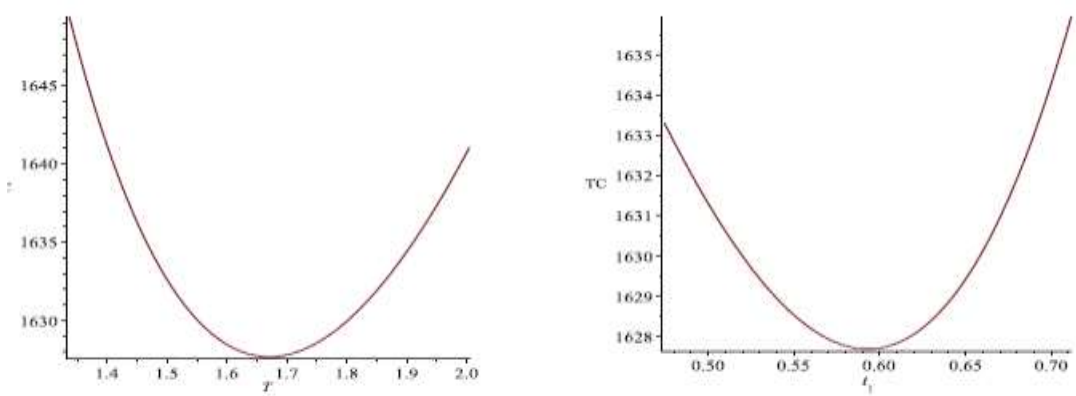
$977.479 > 0$ and $\frac{\partial^2 TC(t_1,T)}{\partial T^2} = 294.161 > 0$,

$\frac{\partial^2 TC(t_1,T)}{\partial T \partial t_1} = -254.961$.Therefore from equation (15), we have:

$\frac{\partial^2 TC(t_1,T)}{\partial t_1^2} * \frac{\partial^2 TC(t_1,T)}{\partial T^2} - \left(\frac{\partial^2 TC(t_1,T)}{\partial T \partial t_1}\right)^2 = 222531.210$. T and t_1 minimizes the total inventory average cost since they both satisfies the necessary and sufficient condition equation (13) and (14) .

When the values of T and t_1 are substituted into equation (10),The total cost $TC(t_1, T) = 1627.689$. and $M = 22.600$, $P = 110.209$ $N = 87.609$

To further establish that the solution are correct,The total cost function is plotted against some values of t_1 and T which gives us a strictly convex graph as shown in Figures 2 below:



(a) Total cost and t_1 (b) Total cost and T

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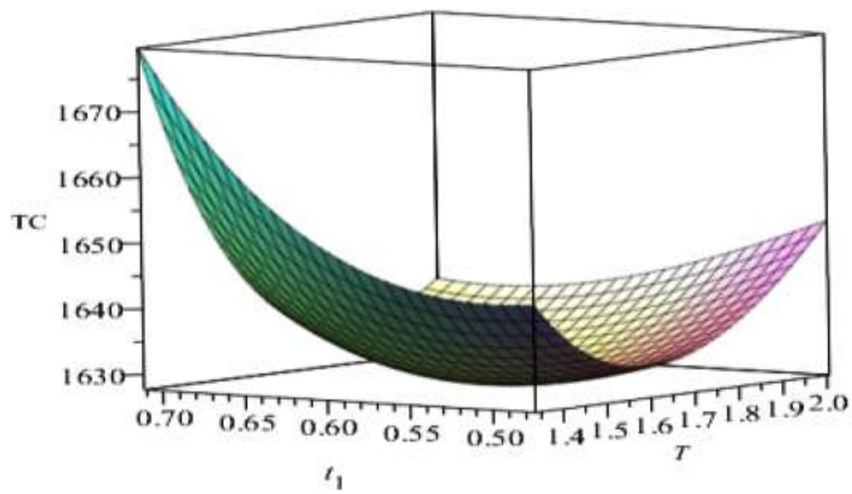

(c) Total cost and (T, t_1)

Fig. 2: Graphical representation of Convexity of total cost per unit time

It is obvious from the Figures 2 that the total cost function is strictly convex function, showing us that the optimum value of t_1 and T can be derived with the aid of total cost function of the policy as long as the inventory total cost per unit time is the minimum.

6. Sensitivity Analysis

The target of every manager is to make best decisions among many options available to them and to review the decision when optimum benefits is not been achieved. In order for such managers to benefit from this model, sensitivity analysis is carried out here to show how this model is effected by the changes in the input parameters.The behaviour of schedule period T, ordering quantity P, and inventory total cost TC against changes in the parameters $d, A, K, S, h, \beta, Z, \theta, \gamma$ and n of the inventory policy are analysed. By varying the values from $+20\%$ to -20% of the individual parameter. One parameter is considered at a time while leaving the other parameters unchanged.

Based on the numerical example given above and the results obtained and analysed in Table 1, the following observations can be derived:

1. Increase in the value of demand rate d leads to decrease in the schedule period T, but there is an increase in the inventory total cost TC and ordering quantity P.
2. It is observed that an increase in ordering cost A leads to increase in the schedule period,inventory total cost and ordering cost.
3. An increase in the shortage cost K results in decrease in schedule period and ordering quantity,but there is an increase in the inventory total cost.

4. Increase in the lost sale S leads to decrease in the schedule period,the inventory total cost and ordering quantity increases.
5. Increase in the value of parameter β and holding cost h results to decrease in the schedule period and ordering quantity, but the inventory total cost increases.
6. Increase in the purchasing cost Z leads to increase in the schedule period and inventory total cost, however there is decrease in the ordering quantity.
7. Increase in the value of deterioration parameter θ leads to decrease in schedule period and ordering quantity, but the inventory total cost is increasing.
8. When the backlogging rate γ is increasing, there is an increase in the inventory total cost and ordering cost which results in decrease in schedule period.
9. Finally, increase in the value of index number n of the power demand pattern results in the increase in the schedule period and inventory total cost,however there is a decrease in the ordering quantity.

Economic implication of the above results are stated thus:

1. Increase in demand rate d result in increase in the total cost TC, ordering quantity$P$, lower cycle time t_1 and T. Implication of this is that increase in demand rate will lead to decrease in the optimal cycle,but results in the higher value of optimal total cost per unit time.This is normal because if the demand rate is higher, the stock will be used up quickly and the cycle time and length will decrease.
2. Increase in the value of the deterioration rate θ result in lower value of cycle length T, lower ordering quantity and increase in the value optimal total cost.The implication of this is that increase in the deterioration rate will lead to decrease in the optimal cycle length.The total cost per unit time will increase because when deterioration cost increases, there will be increase in the inventory total cost per unit time which will lead to stocks getting finished earlier as a result of lower cycle length.
3. Increase in the values of holding cost h and β lead to increase in the value of total cost and decrease in the value of ordering quantity with lower cycle time and length.This is advantageous to retailers in that when the holding cost is kept at minimum, the volume of inventory ordering quantity must be reduce and the time for the stock to used up must also be reduce in order to minimize the inventory total cost.

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ISSN: 2582-6271

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Table 1: Sensitivity Analysis of the Parameters in the Inventory Model

			Change in:						
P*	V*	C*	T	T^*	t_r	$TC(t_r, T)$	U^*	P	G^*
	120	+ 20	1.535	-8.107	0.557	1890.846	16.168	121.791	10.509
	110	+ 10	1.598	-4.328	0.574	1759.868	8.121	116.116	5.360
	100	0	1.671	0	0.593	1627.689	0	110.209	0
d	90	-10	1.755	5.028	0.615	1494.114	-8.206	104.038	-5.599
	80	-20	1.854	10.967	0.639	1358.892	-16.514	97.559	-11.478
	600	+20	1.817	8.781	0.6303	1685.002	3.521	119.613	8.532
	550	+10	1.746	4.537	0.613	1656.953	1.798	115.072	4.412
	500	0	1.617	0	0.593	1627.689	0	110.21	0
A	450	-10	1.597	-4.773	0.572	1597.03	-1.884	102.082	-4.653
	400	-20	1.507	-9.823	0.549	1564.746	-3.867	99.64	-9.591
	12	+ 20	1.545	-7.532	0.609	1674.787	2.894	104.155	-5.486
	11	+ 10	1.603	-4.042	0.601	1650.508	1.402	106.927	-2.971
	10	0	1.671	0	0.593	1627.689	0	110.201	0
K	9	-10	1.750	4.745	0.583	1602.867	-1.525	114.152	3.585
	8	-20	1. 845	10.408	0.572	1575.709	-3.193	118. 967	7.95
	9.6	20	1.658	-0.774	0.617	1683.227	3.412	110.489	0.253
	8.8	10	1.665	-0.356	0.605	1655.559	1.712	110.373	0.045
	8.0	0	1.671	0	0.593	1627.689	0	110.21	0
S	7.2	-10	1.676	-0.305	0.580	1599.624	-1.724	109.998	-0.192
	6.4	-20	1.680	-0.557	0.567	1571.366	-3.460	109.74	-0.426

	0.48	20	1.670	-0.014	0.591	1628.099	0.025	110.112	-0.089
	0.44	10	1.671	-0.007	0.592	1627.895	0.013	110.16	-0.045
	0.4	0	1.671	0	0.593	1627.689	0	110.21	0
h	0.36	-10	1.671	-0.007	0.594	1627.483	-0.013	110.259	0.044
	0.32	-20	1.671	-0.014	0.595	1611.035	-1.023	110.309	0.001
	18	20	1.662	-0.490	0.568	1630.88	0.196	108.821	-1.499
	1.65	10	1.666	0.261	0.580	1629.351	0.102	109.416	-0.720
	15	0	1.671	0	0.593	1627.689	0	110.21	0
β	1.35	-10	1.675	-0.392	0.620	1625.879	-0.111	110.965	0.685
	12	-20	1.681	-0.926	0.647	1625.89	-0.233	111.683	1.337
	14.4	20	1.678	0.429	0.538	1784.485	9.633	108.613	-1.449
	13.2	10	1.675	0.261	0.566	1706.457	4.839	109.416	-0.720
	12	0	1.671	0	0.593	1627.689	0	110.21	0
Z	10.8	-10	1.664	0.392	0.620	1548.107	-4.889	110.965	0.685
	9.6	-20	1.655	0.926	0.647	1467.623	-9.834	111.683	1.337
	0.96	20	1.663	- 0.459	0.572	1630.163	0.152	109.284	-0.840
	0.88	10	1.667	-0.238	0.582	1628.961	0.078	109.734	-0.432
	0.8	0	1.671	0	0.593	1627.689	0	110.21	0
θ	0.72	-10	1.675	0.256	0.604	1626.341	-0.083	110.712	0.455
	0.64	-20	1.680	1.680	0.616	1624.912	-0.171	111.243	0.937
	0.72	20	1.535	-8.10	0.625	1711.88	5.172	119.77	8.674
	0.66	10	1.599	-4.310	0.610	1671.071	2.665	115.253	4.576
Υ	0.60	0	1.671	0	0.593	1627.689	0	110.210	0
	0.54	-10	1.754	4.972	0.573	1581.481	-2.839	104.603	-5.086
	0.48	-20	1.851	10.812	0.550	1532.132	-5.871	98.389	-10.726

7. CONCLUSION

In this model, a linear deteriorating inventory policy for items with variable holding cost and demand assumed to be inform of power demand pattern is proposed. Shortages are allowed and partially backlogged which actually captures real life situation, since some retailers will be willing to patiently wait for arrival of new stock during stock-out, but the longer the waiting time, the possibility of the customers looking for elsewhere to meet their demand.

The objective of this model is to determine the optimal replenishment procedure that minimizes the average inventory total cost per unit time. Optimal order quantity and optimal replenishment cycle time were derived and the solution obtained. The result are further established with the aid of numerical example and sensitivity analysis carried out and depicted graphically of the decision variables with regards to changes in the input parameters in the model.

The results obtained indicate that the effect of power demand index parameter n on the average minimum cost is quiet significant. On thorough examining the effect of the policy input parameters on the decision variables, it was find out that U^* is sensitive to over-estimation and under estimation of the parameters d

and Z while G^* is sensitive to the over-estimation and under-estimation of the parameters d, A, K, n, γ. Also U^* is less sensitive or insensitive to over-estimation and under-estimation of the parameters A, K, n, β, S, h and θ while G^* is less sensitive or insensitive of the parameters n, Z, β, S, h and θ. The proposed model can be extended further by adding quantity discount, trade credit, stochastic demand rate, finite replenishment and so on.

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