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QUANTITATIVE COMPARISONS OF GALILEO'S ERRONEOUS IDEAS ON THE TIDES

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ABSTRACT

Galileo's explanations of tides in terms of Earth rotational and orbital motion around the Sun are erroneous. The actual tidal acceleration due to the differential gravitation by the Moon is calculated to be several orders of magnitude smaller than Earth's rotational, orbital or center of mass motions.

KEYWORDS: History of Science, History of Physics, Tides, Differential Gravitation, Philosophy of Physics, Philosophy of Science

INTRODUCTION

Galileo's explanations of tides in terms of Earth rotational and orbital motion around the Sun are discussed in his Discourse on the Tides in 1616 and Dialogue on the Two Chief World Systems in 1632. In Dialogue, he presents interesting 4-day-dialogues to defend Copernican Sun-centered Universe. During the fourth day of the dialogue in the book, he argued that positive and negative acceleration due to Earth's rotational and orbital motion creates the tides. Here, complete calculations for quantitative comparisons of Galileo's ideas on the tides are given to appreciate Galileo's historical errors concerning the tidal phenomena.

The acceleration due to Earth's rotation

The acceleration due to a circular motion a_c is given by the following expression in any first year college physics text book (Halliday, 2013);

$$a_c = \frac{v^2}{r}$$

, where v is the speed of the circular motion and r is the radius of the circular motion.

First, for the acceleration due to Earth's rotation, it is necessary to express the speed of Earth's rotation in terms of Earth's radius, $r=6371\text{km}$ and its rotational period, $T=24$ hour as follows;

$$v_E = \frac{2\pi r}{T} = \frac{2\pi(6371\text{km})}{24\text{hour}}$$

Therefore, the acceleration due to Earth's rotation a_E can be calculated as follows

$$a_E = \frac{v_E^2}{r} = \frac{\left(\frac{2\pi \times 6371 \times 10^3\text{m}}{24 \times 3600\text{sec}}\right)^2}{6371 \times 10^3\text{m}} = 3.37 \times 10^{-2}\text{m/sec}^2$$

The acceleration due to Earth-Moon circular motion

Second, for the acceleration due to Earth-Moon circular motion, it is necessary to express the speed of Earth's motion relative to the center of mass in Earth-Moon system. The distance from Earth's center to the center of mass, C , is given by the following expression.

$$M_E \times C = M_m \times (D - C)$$

, where $D= 384,400\text{km}$ is the distance from Earth to the Moon, $M_m=7.35 \times 10^{22}$ kg is the mass of the Moon, and $M_E=5.97 \times 10^{24}$ kg is the mass of Earth. Plugging these numbers gives $C=4705\text{km}$ from the center of Earth to the Moon. However, since the radius of Earth is 6371km , thus, the center of mass for Earth-Moon system, C , is located about 1650 km below the surface of Earth.

Since Earth has a relatively small circular motion around the center of the Earth-Moon system and the center of the system is located below the Earth surface, it should be noted that the acceleration due to a circular motion around the center of mass for Earth-Moon system are two kinds; [1] the acceleration on Earth's surface facing toward the Moon and [2] the acceleration on Earth's opposite surface facing away from the Moon.

For the case of [1], the speed of Earth rotation around the center of Earth-Moon system is

$$v_1 = \frac{2\pi r}{T} = \frac{2\pi(1650\text{km})}{27.32\text{days}}$$

, where 27.32 days is the sidereal one month. The sidereal one month is defined to be the Moon's orbital period around Earth relative to distant fixed stars, which is smaller than the synodic one month of 29.53 days defined by the time for one complete cycle of the Moon's full phase. The reason for using the sidereal month is due to the fact that the Moon's orbital period around Earth relative to distant fixed stars is exactly the same as Earth rotational period around the center of Earth-Moon system. In this case, the surface of Earth facing the Moon is rotating around the center of mass for Earth-Moon system which is 1650 km below the surface. Therefore, the circular acceleration on Earth's surface facing toward the Moon can be expressed by

$$a_1 = \frac{v_1^2}{r_1} = \frac{\left(\frac{2\pi 1650 \times 10^3 m}{24 \times 27.32 \times 3600 sec}\right)^2}{1650 \times 10^3 m} = 1.17 \times 10^{-5} m/sec^2$$

For the case of [2], the rotational speed of Earth around the center of Earth-Moon system is

$$v_2 = \frac{2\pi r}{T} = \frac{2\pi(4705 + 6371 km)}{27.32 days}$$

, where 27.32 days is again the sidereal one month. In this case, the opposite surface of Earth facing away from the Moon is rotating around the center of mass for Earth-Moon system which is 6371 + 4705 km below the surface, where, 6371 km is the radius of Earth. Therefore, the circular acceleration on Earth's opposite surface facing away from the Moon is

$$a_2 = \frac{v_2^2}{r_2} = \frac{\left(\frac{2\pi(4705 + 6371) \times 10^3 m}{24 \times 27.32 \times 3600 sec}\right)^2}{(4705 + 6371) \times 10^3 m} = 7.85 \times 10^{-5} m/sec^2$$

The acceleration due to Earth orbital motion

Third, for the acceleration due to Earth orbital motion around the Sun, it is necessary to express the speed of Earth motion relative to the Sun in terms of the distance between the Sun-Earth ($149.6 \times 10^6 km$), the period of Earth motion around the Sun (365.256 days). These give the orbital speed of Earth around the Sun v_{S-E} as follows

$$v_{S-E} = \frac{2\pi r}{T} = \frac{2\pi 149.60 \times 10^9 m}{24 \times 365.256 \times 3600 sec}$$

Therefore, the acceleration due to Earth's orbital motion around the Sun a_{S-E} is

$$a_{S-E} = \frac{v_{S-E}^2}{r} = \frac{(2\pi 149.60 \times 10^9 m / (24 \times 365.256 \times 3600 \text{sec}))^2}{149.60 \times 10^9 km} = 5.93 \times 10^{-3} m/sec^2$$

The actual tidal acceleration

Finally, it is time to discuss the actual tidal force. “The tidal force on Earth due to the Moon arises because of the varying values for the Moon’s gravitational attraction at different locations inside the planet” (p.720, Carroll and Ostler, 2007), Similarly, “It [The tidal force] is due to the difference between the moon’s gravitational pull at the center of the Earth and on the Earth’s surface. “(p.205, Marion and Thornton, 1995)

The tidal acceleration due to the Moon on Earth a_T is given as follows

$$a_T = \frac{GM}{r^3} 2R = \frac{(6.67408 \times 10^{-11})(7.34767309 \times 10^{22} kg)2(6371 \times 10^3 km)}{(384,400 \times 10^3 km)^3} = 1.1 \times 10^{-6} m/sec^2$$

, where G is the gravitational constant, r is the distance from Earth to the Moon, M is the mass of the Moon, and R is the radius of Earth.

CONCLUSION

Therefore, the tidal acceleration due to the differential gravitation by the Moon is several orders of magnitude smaller than Earth’s rotational, orbital or center of mass motions. The following Table summarizes the statement.

Acceleration	m/sec ²
Earth’s rotational	3.37×10^{-2}
Earth-Moon’s center of mass	$1.17 \times 10^{-5} \sim 7.85 \times 10^{-5}$
Earth’s orbital	5.93×10^{-3}
Tidal	1.1×10^{-6}

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